Ideas... problems for tutorials week 19

1. Prove by induction that $6^n - 1$ is divisible by 5 for any $n \in \mathbb{N}$.

2. Suppose that a is a real number such $a + \dfrac{1}{a}$ is an integer. Use Cumulative Induction to prove that then $a^n + \dfrac{1}{a^n}$ is also an integer for every $n = 1, 2, 3, \ldots$.

3. (More difficult problem.) Use induction on n to prove that any 'map' formed by intersections of n straight lines can be coloured in two colours in such a way that no two countries of the same colour have a common boundary segment. (Each country is coloured in one colour; corner points between two countries of the same colour are allowed.)