Algebra – Practical session 2

I would like to remind you to please answer the questions in the given order. There are usually more questions than you will be able to answer in a practical, so take the rest as homework.

However, the best strategy for you, if you find the time, is probably be starting to answer the questions ahead of the practical, so you get rid of the easiest ones and be ready with questions for me in the practical.

Please always read the solutions afterwards, as they often contain alternate solutions or extra comments.

As usual, I have marked with * questions which might be harder than the rest, for example because they require you to think a bit beyond procedures and calculations which we have explicitly practiced in the lectures.

2.1. Use the Euclidean algorithm to compute the greatest common divisor of 89 and 55.

2.2. Use the Euclidean algorithm to compute the greatest common divisor of 89 and 55, but this time allow negative remainders (with $-b/2 < r \leq b/2$, as opposed to the usual condition $0 \leq r < b$ on the remainder).

2.3. Use the extended Euclidean algorithm to find integers s and t such that $89s + 55t = 1$.

2.4.   (a)   Use the Euclidean algorithm to compute the greatest common divisor d of 557 and 337.
   (b)   Find integers s and t such that $557s + 337t = d$.

2.5. Find all integer solutions x, y of $20x + 12y = 6$.

2.6.   (a)   Find all integer solutions x, y of $19x + 12y = 1$.
   (b*)   How many of them satisfy $|x| < 12$ and $|y| < 19$?

2.7*. Express the fraction $\dfrac{1}{77}$ as a difference $\dfrac{a}{7} - \dfrac{b}{11}$, for some *positive* integers a, b.

Now express the same fraction $\dfrac{1}{77}$ as a difference $\dfrac{c}{11} - \dfrac{d}{7}$, for some positive integers c, d.

2.8. In a fictional country the local currency is the 'unit', but there exist only two types of coins (and no banknotes): a 'small coin' worth 13 units, and a 'big coin' worth 17 units. (They love prime numbers over there.)
   Find how to pay for an item worth 7 units (by giving a number of coins to a merchant and possibly receiving a number of coins as change).

2.9*. Same setting of the previous question, in a fictional country the local currency is the 'unit', but there exist only two types of coins (and no banknotes): a 'small coin' worth 13 units, and a 'big coin' worth 17 units.
   Find in exactly how many different ways it is possible to pay for an item worth 700 units, by using any of the two types of coins but without having to receive any change.