Geometrical Optics

Practical 1

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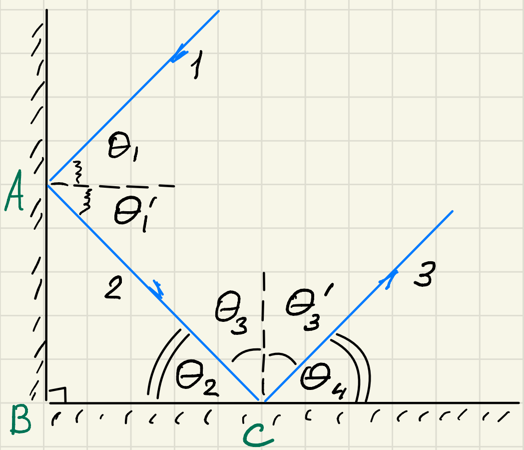
Geometrical Optics

Problems for Week 1

1. Two mirrors are set in the corner perpendicular to each other. A light ray, which travels in the plane perpendicular to both mirrors, hits one of the mirrors. Draw all resulting rays. Choose your own angle of incidence for the first ray (you don't need necessarily to choose a numerical value, you can choose a graphical representation) and find angles of travel for all other rays.

Solution



$\theta_1 = \theta_1'$; $\theta_3 = \theta_3'$

– laws of reflection

Sum of angles in $\triangle ABC$:

$$180^\circ = \theta_2 + 90^\circ + (90^\circ - \theta_1)$$

$$\Rightarrow \quad \theta_1 = \theta_2$$

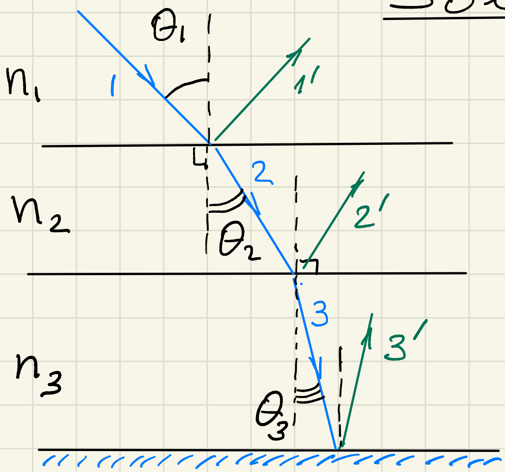
$$\theta_3 = 90^\circ - \theta_2 = 90^\circ - \theta_1$$

$$\theta_4 = 90^\circ - \theta_3' = 90^\circ - \theta_3 = \theta_1$$

$$\boxed{\theta_1 = \theta_4}$$

2. A layer of water is on the top of a horizontal rectangular slab of glass. There is no wind or other perturbances on the surface of water. A light ray hits the water at some non-zero angle with the normal to the water surface. Draw all resulting rays. Choose your own angle of incidence for the first ray (you don't need necessarily to choose a numerical value, you can choose a graphical representation) and find angles of travel for all other rays. The indexes of refraction of air, water and glass are: n_1, n_2, and n_3 correspondingly.

Solution



Assume the glass is on some surface. ( )
(You can try other assumptions too)

n_1 - air
n_2 - water
n_3 - glass

From the lectures:

$$n_1 < n_2 < n_3 \quad (*)$$

Rays 1, 2, 3 - refracted rays, they bend towards the normals due to (*)

Rays 1', 2', 3' - reflected rays

Apply Snell's law for air-water:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad (**)$$

Snell's law for water-glass:

$$\underline{\underline{n_2 \sin\theta_2}} = n_3 \sin\theta_3 \qquad (***)$$

Comparing (**), (***):

$$\boxed{n_1 \sin\theta_1 = n_3 \sin\theta_3}$$

Conclusion: angles in air and glass do not depend on water layer.

Angles for 1', 2', 3' are θ_1, θ_2 θ_3.

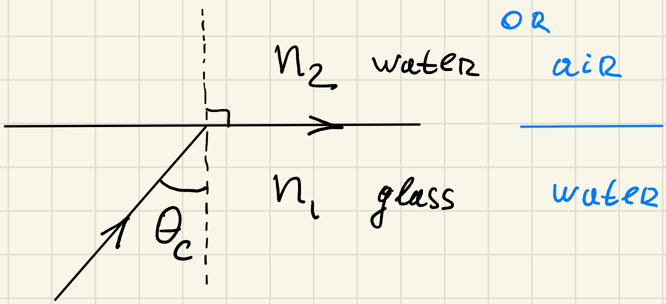
additional questions (try yourself)!

(*) Are rays on the drawing are all rays?

(**) Is it possible to draw all rays?

3. In the setup of the problem 2, light shines from the bottom of glass slab. Find critical angles of total internal reflection at the water-glass interface and the water-air interface. The glass is a crown glass (see the table in the lecture notes).

Solution



The phenomenon is the same for air-water or water-glass interfaces.

From the lectures: $\sin\theta_c = \dfrac{n_2}{n_1}$

n	
1.5	glass
1.33	water
1	air

$(n_1 > n_2)$

$$\theta_c = \sin^{-1}\left[n_2/n_1\right]$$

For glass-water: $\theta_c = \sin^{-1}\dfrac{1.33}{1.5} \simeq$

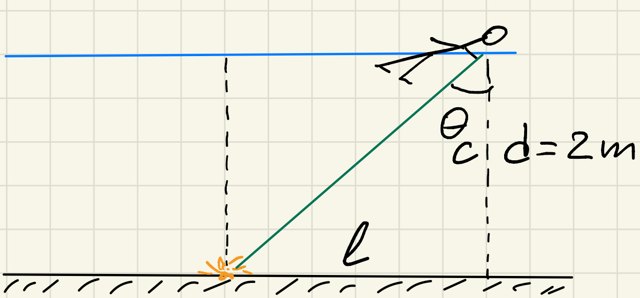
$$\boxed{\simeq 62.5^\circ}$$

For water-air: $\theta_c = \sin^{-1}\dfrac{1}{1.33} \simeq$

$$\boxed{\simeq 48.8^\circ}$$

4. In a middle of the floor of a large 2 m deep swimming pool, there is an electric light. You are swimming in the middle lane (which goes directly above the light source). Will you always see the light looking from the air? Support your answer with the numerical calculations and ignore any waves in the swimming pool.

Solution



Due to the total internal reflection you will not see the light for angles larger than θ_c.

From the previous problem we have:

$$\theta_c = 48.8^\circ$$

Therefore maximum distance l, at which you still see the light is

$$\boxed{l} = d \cdot \tan\theta_c = 2m \cdot \tan 48.8^\circ \simeq$$

$$\simeq \boxed{2.3m}$$