UNIVERSITY OF LINCOLN
SCHOOL OF MATHEMATICS AND PHYSICS

MTH1005M PROBABILITY AND STATISTICS PRACTICAL 4

Short review

Definition 1: Consider an experiment, with outcome set S , split into n mutually exclusive and exhaustive events $E_1, E_2, E_3, \ldots, E_n$. A variable, X say, which can assume exactly n numerical values each of which corresponds to one and only one of the given events is called a random variable.

Definition 2: Let X be a discrete random variable that can take on only the values $x_1, x_2, x_3, \ldots, x_n$ with respective probabilities $p_X(x_1), p_X(x_2), p_X(x_3), \ldots, p(x_n)$.
Then, if

$$\sum_{x \in X} p_X(x) = 1$$

The function $p(x) = P\{X = x\}$ is called the probability (mass) function of the variable X.

Definition 3: The cumulative distribution function of a discrete random variable, X , is

$$F_X(a) = P\{X \leq a\} = \sum_{x \leq a} p_X(x).$$

X is a discrete random variable.

Question 1

Let Z be the 'number around the base when a 4 sided die is rolled'.

- Write down a sample space for the experiment
- Define the values of Z and the events they correspond to.
- Write down the probability mass function of Z.

Question 2

Fully define random variables corresponding to the experiments

- 'number of heads shown when two coins are thrown'.
- 'the number of successes out of 4 trials'.
- an urn contains 3 white and 2 blue balls. Two balls are selected without replacement, and £1 is paid out for each blue ball in the selection.

Question 3

The *cumulative distribution functions (CDF)* of the for the variables in questions 1 and 2 is given by the expression $F(a) = \sum_{\text{all } x \leq a} p_X(x)$. How do the plots of the CDFs look like?

Question 4

Let X be the discrete random variable *'number of tails shown when two coins are thrown'*. Define two more random variables:

$X_1 =$ the number of tails shown on the first coin,

$X_2 =$ the number of tails shown on the second coin.

Show that X, X_1 and X_2 are random variables.

Question 5

In a game 3 dice are rolled. The player bets £1. They get back £1 if they roll a single 5, £2 if 2 fives come up, and £3 if 3 fives come up (and their stake is returned). If no 5s come up they lose their £1 stake.

1. Recalculate the value of the probability mass function that I have shown in the slides/notebook.
2. Calculate the probability mass function for a game with 4 dice.

Question 6

The probabilities that three men hit a target are respectively $\frac{1}{6}$, $\frac{1}{4}$, and $\frac{1}{3}$. Each man shoots once at the target.

1. Find the probability p that exactly one of them hits the target,
2. If only one hit the target, what is the probability that it was the first man?