Computer Algebra with Maple – exercises

Version February 26, 2024

- Finish all the exercises. If you can't complete them in-class, finish them out-of-class.
- Do everything within a Maple `document` file (which has extension `.mw`).
- Store all exercises in your logbook. The Maple document file can be used for this.
- Make regular backups on at least *two* different locations (e.g., OneDrive and email). Note: using a USB stick is not advisable because they can suffer from data corruption. This can happen when you don't do 'eject', but also for other reasons.

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Some of the exercises are based on or from the book *Maple by Example* by Martha L. Abell and James B. Braselton (third edition).

1 Exercises session 1

Note: include all exercises, also the *tutorials*, in your logbook. Do this for this and the subsequent sessions.

1.1 Tutorials

1. Open the *Getting Started* (re-open MAPLE if it is not present any more), then complete the *Tutorial: Talking to Maple* tutorial (first one) and complete this, using a separate blank MAPLE document. Store that document, once finished, on your OneDrive. So you need to reproduce the right column (indicated by 'Results') in a separate MAPLE document.

2. Open the *Getting Started* (re-open MAPLE if it is not present any more), then complete the *Tutorial: Putting Ideas Together* tutorial (second one) and complete this, using a separate blank MAPLE document.

2 Exercises session 2

2.1 Tutorials

1. Open the *Getting Started* (re-pen MAPLE if it is not present any more), then complete the *Tutorial: Commands and Packages* tutorial (third one) and complete this, using a separate blank MAPLE document.

2. Open the *Getting Started* (re-open MAPLE if it is not present any more), then complete the *Tutorial: Plotting* tutorial (fourth one) and complete this, using a separate blank MAPLE document.

2.2 Calculator

Do everything within a MAPLE `document` file (which has extension `.mw`). This can be used for your logbook.

1. (a) Calculate 13×12.
   (b) Calculate $\cos(\pi/2)$. Verify that this is what you would expect.

2. Calculate $n!$ for
   (a) $n = 3$
   (b) $n = 100$
   (c) $n = 200$

3. Calculate and/or simplify
   (a) $(1 + i)(1 - i)$
   (b) $|2 + 4i|$
   (c) i^i
   (d) $\sqrt{i}$

2.3 Calculus

1. Determine the following single or higher order derivatives

(a)
$$f'(x)$$
with
$$f(x) = 8\cos(3x)$$

(b)
$$f''(x)$$
with
$$f(x) = c\sin(\exp(x^3))$$

(c)
$$\frac{\mathrm{d}^3 f(x)}{\mathrm{d}x^3}$$
with
$$f(x) = \cos(x^n) + x^4$$

(d)
$$\frac{\partial^2 f(x,y)}{\partial x \partial y}$$
with
$$f(x,y) = \cos(xy)$$

2. Determine the following integrals

(a)
$$\int x^2 \sin(x)\mathrm{d}x$$

(b)
$$\int_0^x t^2 \sin(t)\mathrm{d}t$$

(c)
$$\int_{-\infty}^{\pi} \exp(y)\sin(y)\mathrm{d}y$$

(d)
$$\int_0^{\pi}\int_1^3 x^2 \sin(y)\mathrm{d}x\mathrm{d}y$$

3. Which of the following integrals can be solved by Maple analytically (i.e., it will return a function) and which other ones numerically? Give the answers if it can be solved either way.

(a)
$$\int \exp(\cos(x))\mathrm{d}x$$

(b)
$$\int \exp(a\cos(bx))\mathrm{d}x$$

(c)
$$\int_0^1 \exp(\cos(x))\mathrm{d}x$$

(d)
$$\int_0^{2\pi} \exp(\cos(x))\mathrm{d}x$$

(e)
$$\int_0^{2\pi} \exp(a\cos(x))\mathrm{d}x$$

(f)
$$\int_0^{2\pi} \exp(a\cos(bx))\mathrm{d}x$$

4. Determine the following sums with Maple

(a)
$$\sum_{k=1}^{10} a$$

(b)
$$\sum_{k=1}^{10} k^2$$

(c)
$$\sum_k k^3$$

(d)
$$\sum_n n^{-2}$$

(e)
$$\sum_{n=1}^{\infty} n^{-2}$$

(f)
$$\sum_{n=0}^{\infty} x^n$$

(g)

$$\sum_{n=1}^{\infty} \frac{1}{n!} x^n$$

5. Determine the following limits with MAPLE

(a)

$$\lim_{x \to 0} \frac{\sin(8x)}{2x}$$

(b)

$$\lim_{x \to \infty} x \exp(-x)$$

(c)

$$\lim_{x \to \infty} \frac{x - 8x^4}{7x^4 + 5x^3 + 2000x^2 - 6}$$

(d)

$$\lim_{x \to \infty} \frac{\sqrt{16x^4 + 8} + 3x}{2x^2 + 6x + 1}$$

(e)

$$\lim_{x \to 0} \frac{\cos(ax) - 1}{bx^2}$$

(f)

$$\lim_{x \to 0} \frac{\exp(x) - 1}{|x|}$$

(g)

$$\lim_{x \to 0^+} \frac{\pi x}{|x|}$$

(h)

$$\lim_{x \to 0^-} \frac{x}{c|x|}$$

Verify the limits graphically by plotting the expression around the limit value (substitute numerical values for variables if needed).

3 Exercises session 3

Do everything within a MAPLE `document` file (which has extension `.mw`). This can be used for your logbook.

3.1 Solving equations

1. Solve the following equations

(a)

$$3x^2 + 4 = x$$

for x

(b)

$$\exp(x^2) - \cos(c^2 + 3c + 4) = 0$$

for x. Determine the value of c so that x has a root equal to 0

3.2 Taylor series

1. Determine the Taylor series of the following functions (consult the MAPLE help on `taylor` and `taylor/details` if needed)

(a)

$$\exp(x)$$

around $x = 0$ with order 3. A Taylor series around $x = 0$ is also known as a MacLaurin series.

(b)

$$\exp(x)$$

around $x = 1$ with order 4.

(c)

$$\sin(\cos(\tan(x)))$$

around $x = 0$ with order 5.

around $x, y = 3, 1$ with total order 2

(d)

$$\sin(x)\cos(y)$$

3.3 Variables and functions

1. Define a variable `myage` and assign it the value of your age. Subtract two from the value of the variable, and update the variable `myage` with this new value. Add one to the value of the variable, and update the variable again. Observe the Variables pallete as you do this.

2. Define and assign a variable L equal to 6, W equal to 10, and `area` equal to $L \cdot W$. What is the value of `area`? Finally, remove all assignments for L, W and `area`.

3. (a) Define a function $f(x) = 8\cos(3x)$ in MAPLE. Calculate the derivative of this function, $f'(x)$. (i.e., instead of directly inputting the function within the derivative, as you may have done in exercise 2.3.1a).

   (b) By using a function definition, also calculate the other derivatives as given in the remaining questions of the same exercise (also instead of using the direct method).

4. The derivative of a function $f(x)$ can be defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   Calculate the derivative of $f(x) = \sin(2x)$ using

   (a) this limit

   (b) the normal way

   Plot both results and compare. Also calculate the difference between the two expressions. As always, simplify if needed.

5. Consider the function

$$f(x) = \exp(\cos(x))$$

   (a) Calculate the 7th order Maclaurin series.

   (b) Convert the resulting expression to a polynomial. (consult MAPLE taylor/details help if needed)

   (c) Plot both the original function and the polynomial in the same graph.

   (d) Integrate the polynomial from 0 to 1.

   (e) Compare the value of the previous integral with the numerical result of the integral of $\exp(\cos(x))$ over the same range.

   (f) Calculate the %ge error in the integral for the 5th order and the 7th order.

3.4 Linear algebra

1. Hint: to solve the following exercises, you can consult the help for the relevant command and in particular the examples. Pay attention to the package that sometimes needs to be loaded using `with` (the package name, if needed, is mentioned in grey above the name of the command in the MAPLE help).

   (a) Look up the help for `PlanePlot`. What is the text in grey before the word **PlanePlot**?

   (b) Load the package for `PlanePlot` by typing `with(Student[LinearAlgebra])`

   (c) Plot the plane $x+y+2z=1$ using `PlanePlot`. Verify on the plot the expected crossings with the axes.

   (d) Plot the plane $x+y+2z=1$ using `implicitplot3d`. Verify on the plot the expected crossings with the axes.

   (e) Plot both the plane $3x-y+2z=5$ and $x+4y-z=2$ in a single plot using `implicitplot3d`.

2. Let
$$A = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

   (a) Compute with MAPLE A, AA, A^3, A^4, A^5, A^6, A^7. What do you notice?

   (b) Compute A^{2024}, A^{2025}.

   (c) Compute A^{-1}, and $A^{-1}A$ and AA^{-1}.

3. Let
$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
   Compute B^{-1}, $B^{-1}B$ and BB^{-1}. Simplify the results if possible.

4 Exercises session 4

Do everything within a MAPLE `document` file (which has extension `.mw`). This can be used for your logbook.

4.1 Matrix elements

1. Open the *Getting Started* (re-open MAPLE if it is not present any more), then complete the whole of *Tutorial: Working with Matrices* tutorial (fifth one), using a separate blank Maple document.

2. Enter the following identity matrix using `IdentityMatrix`:
$$I_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Define `mb` to be the matrix
$$\begin{pmatrix} 10 & -6 & -\sqrt{3} \\ 6i+5 & y-5 & -7x^2 \\ -10 & 9-i & \pi \end{pmatrix}$$

(a) Extract the third row of `mb`.

(b) Extract the second column of `mb`.

(c) Extract the element in the first row and third column of `mb`.

(d) Extract the element from the last row and last column, using the -1 notation.

(e) Set the first element of the matrix to `x`

(f) Set the second column vector of the matrix to

$$\begin{pmatrix} 15 \\ y+5 \\ z \end{pmatrix}$$

4.2 Vectors

1. Let

$$v = \begin{pmatrix} 0 \\ 5 \\ 1 \\ 2 \end{pmatrix}$$

$$w = \begin{pmatrix} 3 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

(a) Calculate $v - 2w$ and $v \cdot w$.

(b) Find a unit vector with the same direction as v and one with the same direction as w.

2. (a) Using MAPLE, find the angle in degrees between the vectors

$$u = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$$

and

$$v = \begin{pmatrix} -4 \\ 3 \\ -2 \end{pmatrix}$$

(b) Plot both vectors into one graph.

4.3 Sequences and lists

1. (a) Open the *Getting Started* (re-open MAPLE if it is not present any more), then complete the *Sequences*, *Sets* and *Lists* sections of the *Tutorial: Data Structures* tutorial (sixth one), using a separate blank Maple document.

(b) What is the difference between a *list* and a *sequence*?

(c) What is the difference between a *list* and a *set*?

2. Solve the following by making use of the `seq` command.

(a) Generate a sequence of integers with step 1 from 0 up to and including 10.

(b) Generate a sequence of integers from 0 to 30 with step 2.

(c) Generate a sequence 0, 1, 2^2, 3^2, ..., 12^2.

(d) Generate a *list* with content ranging from -1, 0, 1, 2, ... 8.

(e) Generate a sequence x, x, ..., x (100 times).

(f) Differentiate the function x^{200} 100 times using the previous sequence.

(g) Generate a sequence x^2, x^2+1, x^2+2, ..., x^2+5.

(h) Turn the previous sequence into a list.

(i) Plot this list, for x in the range $[0, 4]$.

3. (a) Plot a circle $x^2+y^2=r^2$ for $r=2$ using `implicitplot`.

(b) Plot the circles $x^2+y^2=r^2$ for $r=1$, 2, 3 and 4 within one plot using `implicitplot` and `seq`.

(c) Plot the circles $(x-a)^2+y^2=2^2$, for $a=-8$, -4, 0, 4, and 8. Use the same commands. What happens if you provide the option `scaling=constrained` to the plotting command? And what if you also add `numpoints=10000`?

(d) Plot the circles $(x-a)^2+(y-b)^2=2^2$, for $a, b=-8$, -4, 0, 4, and 8. Use the same commands and extra options.

5 Exercises session 5

5.1 Vectors and matrices

1. The projection vector of a vector $\vec{u}$ onto $\vec{v}$ is given by

$$P_{\vec{v}}(\vec{u}) = \frac{\vec{u}\cdot\vec{v}}{||\vec{v}||^2}\vec{v}$$

where $||\vec{v}||$ is the norm of $\vec{v}$.

(a) Calculate $P_{\vec{v}}(\vec{u})$ for

$$\vec{u} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

and

$$\vec{v} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

(b) Calculate the vector that is perpendicular to the projection vector and $\vec{v}$, $Q_{\perp\vec{v}}(\vec{u}) = \vec{u} - P_{\vec{v}}(\vec{u})$.

(c) Visualize the previous four vectors, using `arrow` for each vector. Combine the plots using `display`. Use different colors for each vector. Also use the option `scaling=constrained`.

2. Set

$$A = \begin{pmatrix} 7 & 3 \\ 4 & 4 \end{pmatrix}$$

and

$$B = \begin{pmatrix} a^2+6 & \sqrt{a^2+i^2}+3 \\ b-3 & 4b/7 \end{pmatrix}$$

Furthermore, set $a=-1$ and $b=7$. By using `EqualEntries` determine whether the two matrices are equal to each other.

5.2 Sequences (2)

1. (a) By using the command `seq` generate the sequence 1, 3/2, 2, 5/2, 3, 7/2, 4. Store the result in the variable `seqone`.
   (b) Set the variable `first` equal to the first element of the previous sequence, using index notation.
   (c) Instead of a sequence, create a list with the same elements and call it `listone`
   (d) Set the variable `last` equal to the last element of the previous list, using index notation.
   (e) From a *list* one can create a *sequence* using `op`. Create a sequence `seqtwo` from the list `listone`.

2. (a) Create a list that contains sin(1), sin(2), ..., sin(10). Use `seq`.
   (b) Create a list that contains the first 25 primes and store the result in a variable. Use the command `ithprime`.
   (c) What is the 15th prime number? Use index notation.

3. (a) Create a sequence π, 1, by just using a comma and the (symbolic) numbers.
   (b) Create a sequence `s1` containing the numbers 1, 2, ..., 10.
   (c) Create a sequence `s2` containing the numbers 9, 8, ..., 1.
   (d) Create a sequence `s3` containing the numbers 10, 9, 8, ..., 1. The sequence should use `s2`, by prepending it with the number 10.
   (e) Create a sequence `stot` containing the numbers 1, 2, ..., 10, 9, 8, ..., 1. This sequence should use the sequences `s1` and `s2`.
   (f) Create a list `l1` containing the same numbers as `s1`.
   (g) Create a list `l2` containing the same numbers as `s2`.
   (h) Combine the two previous lists into a new list `ltot` containing the same numbers as `stot`. Hint: make use of `op`, then combine the sequences and finally convert it back to a list.

4. (a) Create the following list of lists: [[1,2], [3,4]].
   (b) Create the list of lists: [[a,b],[c,d]]
   (c) Convert the previous list into a matrix using `Matrix`.
   (d) Calculate the inverse of the previous matrix.
   (e) Create the list of lists: [[0,1],[1,2],[2,0]] and plot it. Use `style=point`.
   (f) Create the list of lists: [[1, sin(1)], [2, sin(2)], ..., [1000, sin(1000)]]. Plot it using the `point` style.
   (g) Do the same for the list of lists: [[1/100, sin(1/100)], [2/100, sin(2/100)], ..., [10, sin(10)]].

5. (a) Define the function $f(x) = \sqrt{x}$. Create the list $[f(1), f(2), \ldots, f(100)]$, and set the variable `lnat` to this list.
   (b) Create the list `lodd` from the list `lnat`, using `seq`. This new list should contain the odd elements of the old list. Hint: use index notation.

6. (a) Construct the list [1, x, x^2, ..., x^{10}].
   (b) Construct the list $[\frac{d}{dx}1, \frac{d}{dx}x, \ldots, \frac{d}{dx}x^{10}]$ using `seq` and `diff`.

Extra exercises

If you finished all the exercises in-class and updated your logbook too, then you can do these extra exercises to fill up the remaining time of the in-class session.
Do everything within a MAPLE `document` file (which has extension `.mw`). These extra exercises could be used for your logbook.

1. Open the *Getting Started* (re-open MAPLE if it is not present any more), then complete all remaining tutorials.

2. Complete the exercises as given in the eBook *Maple by Example* by Martha L. Abell and James B. Braselton (third edition). The library has an online copy, which you can access via `library.lincoln.ac.uk`.