MTH1002M Calculus **Practical 2**

1. Determine whether the following functions are odd, even, or neither:

   a) $f(x)=\dfrac{2x}{3x^2+1}$ b) $f(x)=\dfrac{3x^2}{2x^4+4}$ c) $f(x)=1+x^3-x^5$

   d) $f(x)=\ln(2x^6-x^4+2)$ e) $g(t)=\dfrac{2t^3+t-1}{3t^2+5}$

   f) $h(x)=\dfrac{ax^3+bx}{cx^2+d}$, where a, b, c and d are non-zero constants.

2. Use the definitions of the hyperbolic functions to show that $\cosh^2 x-\sinh^2 x=1$. Hence show that $\operatorname{sech}^2 x=1-\tanh^2 x$ and that $\operatorname{csch}^2 x=\coth^2 x-1$.

3. Calculate the following powers of $i=\sqrt{-1}$:

   (a) i^2 (b) i^3 (c) i^4 (d) i^5
   (e) i^{100} (f) i^{505} (g) i^{-1} (h) i^{-7}

4. Two complex numbers, z_1 and z_2, are given by $z_1=-2+i$ and $z_2=1-3i$. Evaluate the following:

   (a) $\operatorname{Re}(z_1)$ (b) $\operatorname{Im}(z_1)$ (c) $\operatorname{Re}(z_2)$ (d) $\operatorname{Im}(z_2)$
   (e) $|z_1|$ (f) $|z_2|$ (g) $\arg(z_1)$ (h) $\arg(z_2)$

5. Two complex numbers, z_1 and z_2, are given by $z_1=3-2i$ and $z_2=1+4i$. Calculate the following quantities, plotting each result on an Argand diagram:

   (a) z_1+z_2 (b) z_1-z_2 (c) z_1z_2
   (d) z_1/z_2 (e) z_2/z_1 (f) $\bar{z}_1\bar{z}_2$

6. Write the following complex numbers in polar form:

   (a) $1+i$ (b) $-1+i$ (c) $-1-i$ (d) $\sqrt{3}+i$ (e) $\sqrt{3}-i$ (f) $-1-\sqrt{3}i$

7. Express the following complex numbers in standard form $(z=a+ib)$:

   (a) $\dfrac{1+i}{i(2-3i)}+\dfrac{2}{i}$ (b) $\dfrac{1}{3+2i}+\dfrac{1}{2-i}$

8. Show that the product of two odd functions, $f(x)$ and $g(x)$, is always an even function.

Appendix: Table of values of trigonometric functions.

	0	***π*/6**	***π*/4**	***π*/3**	***π*/2**
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	*