

1. Write down the domain and range of the following functions:

- a) $f(x) = x^2$ b) $f(x) = x^5$ c) $f(x) = \sqrt{5 - x^2}$ d) $f(x) = 4\sqrt{x}$
- e) $f(x) = \frac{1}{x^5}$ f) $f(x) = \frac{1}{x^4}$ g) $f(x) = \frac{1}{(x-3)}$ h) $f(x) = \frac{1}{\sqrt{4 - x^2}}$
- i) $f(x) = \sqrt{a - x}$, where a is a constant j) $f(x) = \frac{1}{(x-b)^2}$, where b is a constant

2. Write down the degree of the following polynomials:

- a) $y = x + 1$ b) $y = 1 + x + 3x^3$ c) $y = 3 + 2x^7 + 5x^2 + x^4$ d) $y = \pi$

3. Sketch the graph of $y = \cos x$ for $-2\pi < x < 2\pi$, and use your result to help you sketch a graph of $y = \sec x = 1/\cos x$ over the same interval. Clearly label your axes in both cases. State the domain and range of $y = \cos x$ and $y = \sec x$.

4. Sketch the graph of the function $f(x) = x^2 - 2x + 1$, and state its domain and range.

5. In the lectures, we saw that a circle in the (x, y) plane is not the graph of a function. We were able to define two functions, $y = \sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$, to represent the top and bottom of the circle respectively. Is it possible to define other sets of functions to represent the circle? If so, give an example, both as a sketch and as a set of formulas.

6. What happens to the function $G(x) = (x^2 - 4)/(x + 2)$ when $x = -2$? Should this point be included in the domain?

7. Write down the first three terms in the power series definition of $\cos x$ given in the lecture notes and confirm that differentiating these gives the first two terms in the power series of $-\sin x$.

8. Use the trigonometric identity $\sin^2 x + \cos^2 x \equiv 1$ to show that

(a) $\cot^2 x + 1 \equiv \operatorname{cosec}^2 x$

(b) $\frac{2\sin x}{1 - \cos x} - \frac{2\cos x}{\sin x} \equiv 2 \operatorname{cosec} x$

9. In the lectures, we saw the general form of the graphs of $f(x) = 1/x^n$ for both odd and even n . How do these graphs change as n increases?

10. Write the power series definitions of $\sin x$ and $\cos x$ given in the lectures in sigma notation i.e. an expression involving a sum from 0 to ∞ written as $\sum_{n=0}^{\infty}$.