

Problems for Tutorials: Week 20

Problem 1. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard vectors in \mathbb{R}^3 . Determine whether

$$\text{span}(\mathbf{e}_1, -2\mathbf{e}_2, \mathbf{e}_1 + \mathbf{e}_2 - \mathbf{e}_3) = \mathbb{R}^3 \quad .$$

Problem 2. Find the span of the following vectors:

$$(i) \quad \mathbf{u} = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad (ii) \quad \mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ -1/2 \\ 3/2 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Problem 3. Find a value for c so that the following vectors:

$$(i) \quad \mathbf{u} = \begin{bmatrix} 1 \\ 1-c \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

are linearly independent.

Problem 4. Find the dot product, the length and the angle between the vectors:

$$\mathbf{u} = \begin{bmatrix} 0 \\ 1/2 \\ -1/2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

Verify the Cauchy–Schwarz inequality and the triangle inequality.