Practical 6

Problem 1. Show that

$$S = \left\{ \begin{bmatrix} x \\ x-y \\ x+2y \end{bmatrix} : x, y \text{ in } \mathbb{R} \right\}$$

is the vector subspace of $\mathbb{R}^3$ and find a basis for S.

Problem 2. Which of the following sets are vector subspaces of $\mathbb{R}^3$ and why.

(a) $S_1 = \{\mathbf{x} \text{ in } \mathbb{R}^3 : x_1 = x_2 = x_3\}$

(b) $S_2 = \{\mathbf{x} \text{ in } \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}$

(c) $S_3 = \{\mathbf{x} \text{ in } \mathbb{R}^3 : x_1 \geq 0\}$

Problem 3. Find a basis for the set of solutions of the following linear system:

$$\left\{ \begin{aligned} x_1 + x_2 + 3x_3 - x_4 &= 0 \\ x_2 + 4x_3 + x_4 &= 0 \end{aligned} \right.$$

Problem 4. Check whether the following vectors form a basis for $\mathbb{R}^3$

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}.$$

Problem 5. Find $col(A), row(A)$ and $null(A)$ for the matrix:

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}.$$

Find a basis for each for the above vector subspaces and determine their dimension. Verify the Rank Theorem: $rank(A) + nullity(A) = dim(\mathbb{R}^3\ )$.