Tutorial Week 4 **Complex numbers and limits**

1. Suppose that $z_1 = 2 + i$, $z_2 = 3 - 2i$ and $z_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$. Evaluate the following:

   (a) $\left|3z_1 - 4z_2\right|$ (b) $z_1^3 - 3z_1^2 + 4z_1 - 8$ (c) $\left(\overline{z_3}\right)^4$ (d) $\left|\frac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|^2$

2. Two-dimensional vectors can be represented by complex numbers. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ represent two non-collinear or non-parallel vectors. If *a* and *b* are real numbers (scalars) such that $az_1 + bz_2 = 0$, show that *a* = 0 and *b* = 0.

3. Find the equation of a circle in the Argand diagram with radius 4 and its centre at (-2, 1).

4. Express $e^{1-i\pi/2}$ in the form $a + ib$.

5. For two complex numbers z_1 and z_2, show that

   (a) $\left|z_1 + z_2\right| \le \left|z_1\right| + \left|z_2\right|$ (b) $\left|z_1 + z_2 + z_3\right| \le \left|z_1\right| + \left|z_2\right| + \left|z_3\right|$

   and give a graphical interpretation of these inequalities.

6. If $f(x) = \frac{x^2 - 3x + 2}{x - 2}$ and $g(x) = x - 1$, for which values of *x* is the statement

$$f(x) = \frac{x^2 - 3x + 2}{x - 2} = \frac{(x-2)(x-1)}{x-2} = x - 1 = g(x)$$

   valid? What are the domains of $f(x)$ and $g(x)$?

7. Is it possible for the statement $\lim_{x \to 2} f(x) = 5$ to be true if $f(2) = 3$? If so, can you think of an equation for which this is the case?

8. Explain what is meant by $\lim_{x \to 1-} f(x) = 3$ and $\lim_{x \to 1+} f(x) = 7$. If both these statements are true, does $\lim_{x \to 1} f(x)$ exist?

9. The signum function is given by

$$\operatorname{sgn} x = \begin{cases} -1 \text{ if } x < 0 \\ 0 \text{ if } x = 0 \\ 1 \text{ if } x > 0 \end{cases}$$

   (a) Sketch the graph of this function.

   (b) State each of the following limits, or explain why it does not exist.

   (i) $\lim_{x \to 0+} \operatorname{sgn} x$ (ii) $\lim_{x \to 0-} \operatorname{sgn} x$ (iii) $\lim_{x \to 0} \operatorname{sgn} x$ (iv) $\lim_{x \to 0} \left|\operatorname{sgn} x\right|$