Algebra – Tutorial week 3

T3.1. Find all the integer solutions of each of the following equations

(a) $33x + 26y = 5$;
(b) $33x + 24y = 16$;
(c) $34x + 24y = 14$.

T3.2. (a) Find all integer solutions x, y of $19x + 12y = 200$.
(b*) How many of them satisfy both $x \geq 0$ and $y \geq 0$? Find them.

T3.3*. The following statement about integers looks similar to a Lemma we saw in the lectures (to explain why the Euclidean algorithm works), but is false: *if $a = bq + c$ and d divides both a and c, then d divides b.*

Show that the statement is false by finding a *counterexample.* That is, find integers a, b, c, d, q which satisfy the hypotheses, but such that the conclusion (*then d divides b*) is not satisfied.