**Tutorial No. 7** **Differentiation, inverse functions and tangents**

1. Differentiate the following with respect to $x$:

   (a) $y = \exp(\sin^2 5x)$ (b) $y = \ln\left[\dfrac{\cosh x - 1}{\cosh x + 1}\right]$ (c) $y = \ln\left[e^x \left(\dfrac{x-2}{x+2}\right)^{3/4}\right]$

   (d) $y = \ln\left(x^2\sqrt{1-x^2}\right)$ (e) $y = \dfrac{e^{2x}\ln x}{(x-3)^3}$.

2. Show that $x = Ae^{-kt}\sin pt$ is a solution of the equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + (p^2 + k^2)x = 0$$

   What would the graph of $x(t)$ look like for $k > 0$ , $k = 0$ and $k < 0$?

3. (a) Find the equation of the tangent to the parabola $y^2 = 4px$ at the point $(x_0, y_0)$.
   (b) Find the $x$-intercept of this tangent.
   (c) Find the equation of the normal to the same parabola at $(x_0, y_0)$, and find its $x$-intercept.

4. Find the expression for the slope at all points on the lower half of the circle $x^2 + y^2 = 25$.

5. Find the slope at a general point on

   (a) the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (b) the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

6. Use implicit differentiation and trigonometric identities to show that

   (a) $\dfrac{d}{dx}\left(\sin^{-1} x\right) = \dfrac{1}{\sqrt{1-x^2}}$ (b) $\dfrac{d}{dx}\left(\tan^{-1} x\right) = \dfrac{1}{1+x^2}$

   Note: in answering this question, it may be helpful to produce rough sketches of the inverse trigonometric functions, explaining how these can be generated from the graphs of the trigonometric functions by reflecting these in the line $y = x$.