Tutorial No. 5 **Limits**

1. L'Hôpital's rule states that

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)},$$

where $f'(x)$ is the derivative $\mathrm{d}f(x)/\mathrm{d}x$. In the lectures and problem classes, we only applied this rule to cases where a was finite. By making a suitable change of variable, show that this formula can also be used to find limits as $x\to\infty$.

2. To evaluate

$$\lim_{x\to 0}\frac{3x^2-1}{x-1}$$

by l'Hôpital's rule, we differentiate the numerator and denominator to obtain $6x/1$, and then substitute $x = 0$. However, if we look at the original function, we see that as x approaches 0 the function approaches 1. Why does this discrepancy occur?

3. Evaluate the following limits:

(a) $\lim_{x\to 0}\frac{\cot x}{\cot 2x}$ (b) $\lim_{x\to\infty}\frac{x^2}{e^x}$.

In part (b), can you say what would happen for higher powers of x?

(c) $\lim_{x\to\pi/2}\frac{\sec x+1}{\tan x}$ (d) $\lim_{x\to\infty}\frac{a_0x^n+a_1x^{n-1}+\cdots+a_n}{b_0x^n+b_1x^{n-1}+\cdots+b_n}$, for n a positive integer.

(e) $\lim_{x\to\infty}\frac{a_0x^m+a_1x^{m-1}+\cdots+a_m}{b_0x^n+b_1x^{n-1}+\cdots+b_n}$, for $m < n$, m and n positive integers.

(f) $\lim_{x\to 0}x\cot x$ (g) $\lim_{x\to\pi/2}\left(x-\frac{\pi}{2}\right)\tan x$ (h) $\lim_{x\to 0+}x\ln x$

4. Using the sandwich theorem, show that $\lim_{x\to 0}x^2\sin\frac{1}{x}=0$.

5. Evaluate $\lim_{x\to\infty}\frac{\sqrt{4x^2+8}+3}{5x+3}$.

6. Find and classify the discontinuities of the following functions:

(a) $f(x)=\frac{a}{x}$ where a is a nonzero constant (b) $f(x)=\frac{x}{(x+4)(x-1)}$

(c) $f(x)=\frac{x^3-27}{x^2-9}$ (d) $f(x)=[x]=$ the greatest integer $\le x$

(e) $f(t)=\begin{cases}0 \text{ if } t=0\\ 3 \text{ if } t\neq 0\end{cases}$ (f) $f(x)=\begin{cases}x & \text{if } x\le 0\\ x^2 & \text{if } 0<x<1\\ 2-x & \text{if } x\ge 1\end{cases}$