

1. Find the critical points of the following functions:

(a) $f(x) = x^{7/8}(x - 3)^2$ (b) $f(x) = \frac{x - 1}{x^2 + 4}$

2. Find the absolute maxima and minima of the following functions:

(a) $f(x) = 2x^3 + 3x^2 - 12x - 12$ on $[-3, 2]$ (b) $f(x) = x + \frac{1}{x}$ on $[0.2, 4]$

3. Find the intervals of increase and decrease of the following functions:

(a) $f(x) = x^3 - 3x + 2$ (b) $g(x) = x \exp(-3x^2/2)$ (c) $G(x) = x + \frac{4}{x^2}$

4. Find a formula for $\tanh^{-1} x$ and state the domain and range of this function.

5. In the lectures, we saw that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$. Use this result to show that

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1 + x^2}}.$$

6. Show that, of all rectangles with area A , the square has the smallest perimeter.

7. Suppose that a rectangular plot is to be fenced off. One side of the rectangle lies along a river, so that no fencing is needed. What are the dimensions of the maximum area that can be fenced in with L metres of fencing?

8. Find the absolute maximum and minimum values of the following functions:

(a) $f(x) = x^3 - x^2 - 8x + 1$ on $[-2, 2]$ (b) $g(x) = x^5 + x + 1$ on $[-1, 1]$
(c) $F(x) = \frac{x + 1}{x^2 + 1}$ on $[-1, 1/2]$

9. Find the intervals of increase and decrease of the following functions:

(a) $F(x) = x^2(x - 5)^3$ (b) $h(x) = \frac{x^2 - 2x + 2}{x - 1}$