MTH1002 Calculus **Practical 5**

1. Differentiate the following functions with respect to x:

   (a) $f(x) = 3x^5 - 2x^2 + \frac{7}{\sqrt{x}} + 2$ (b) $f(x) = \frac{x^3 + 3x^2 + 7}{x - 1}$ (c) $f(x) = e^x \sin x$

   In parts (d)–(f), a, b and c are non-zero constants.

   (d) $f(x) = 4x^4 e^{cx}$ (e) $f(x) = \frac{\cos ax}{x^3}$ (f) $f(x) = \frac{ae^{bx}}{\sin cx}$

2. Differentiate the following functions with respect to x:

   (a) $f(x) = \sin(5x + 2)$ (b) $f(x) = \ln\left(1 + \frac{1}{x^2}\right)$ (c) $f(x) = \sin^3 x$

3. Differentiate the following functions with respect to t:

   (a) $h(t) = (t^3 - 1)^{100}$ (b) $h(t) = \sin(a + bt^4)$ (c) $h(t) = a\cos(b\tan ct)$

4. Use ***logarithmic differentiation*** to find

   (a) $\frac{d}{dx}(x^4 e^{3x} \tan x)$ (b) $\frac{d}{dx}\left(\frac{e^{4x}}{x^3 \cosh 2x}\right)$

5. From first principles (i.e., starting from the definition $f'(x) = \lim_{h \to 0}[f(x + h) - f(x)]/h$), differentiate the following functions with respect to x:

   (a) $f(x) = ax^3 + bx$ (b) $f(x) = \sqrt{cx + d}$ (c) $f(x) = \frac{1}{\sqrt{ex + f}}$

   where a, b, c, d, e and f are non-zero constants.

6. If $f(x) = (x - a)(x - b)(x - c)$, where a, b and c are constants, show that

$$\frac{f'(x)}{f(x)} = \frac{1}{x - a} + \frac{1}{x - b} + \frac{1}{x - c}.$$

7. (a) Differentiate the double-angle formula $\cos 2x = \cos^2 x - \sin^2 x$ with respect to x to obtain the corresponding formula for the sine function.
   (b) Differentiate the addition formula $\sin(a + b) = \sin a \cos b + \cos a \sin b$ with respect to a to obtain the corresponding formula for the cosine function.