MTH1002 Calculus **Practical 4**

1. Evaluate the following limits:

   (a) $\lim_{x\to 0} \dfrac{x^2+2x+1}{x+1}$ (b) $\lim_{x\to 2} \dfrac{x^2-5x+6}{x^2-4}$

2. Evaluate the following limits:

   (a) $\lim_{x\to 0} x\cos x$ (b) $\lim_{x\to 0} \dfrac{\sin x}{e^{2x}-e^{-2x}}$ (c) $\lim_{x\to 0} \dfrac{1-\cos 2x}{xe^x-x}$

   (d) $\lim_{y\to 0} \dfrac{\tan y}{\tan ay}$, where a is a non-zero constant. (e) $\lim_{t\to 5} \dfrac{2-\sqrt{t-1}}{t^2-25}$

3. Evaluate

   (a) $\lim_{x\to\infty} \dfrac{x^4-2x^3+3x^2+5}{4x^4-1}$ (b) $\lim_{v\to\infty} \dfrac{5v^9-4v^5+3v-21}{-2v^9+v^8-4v^2+3v}$

   (c) $\lim_{u\to\infty} \dfrac{u^3-3u^2+5}{u^4+5u^3+1}$

4. For which values of x are the following functions continuous? Give a brief explanation in each case. You may assume that trigonometric functions, polynomials and rational functions are continuous on their respective domains.

   (a) $f(x)=x\sin\left(\dfrac{1}{x}\right)$ (b) $F(x)=\dfrac{x^2-3x+2}{x-2}$ (c) $g(x)=\tan x$

   (d) $G(x)=\cos(x^3+2x^2+3)$ (e) $h(x)=x^{500}-2x^6+80$

   (f) $H(x)=\dfrac{x^2+2x+3}{x^2-1}$

5. Use the sandwich theorem to show that

$$\lim_{x\to\infty} \frac{\sin x}{x} = 0.$$

6. Evaluate the following limits:

   (a) $\lim_{x\to 0}\left(\dfrac{1}{\sin x}-\dfrac{1}{x}\right)$ (b) $\lim_{\theta\to(\pi/2)^-}(\sec\theta-\tan\theta)$ (c) $\lim_{h\to 0} \dfrac{\sqrt[5]{32+h}-2}{h}$