MTH1002 Calculus **Practical 3**

A table of values of sin, cos and tan is overleaf

1. Write the following complex numbers in exponential form:

   (a) $-i$ (b) $-\frac{1}{2}-\frac{i}{2}$ (c) $-\sqrt{3}+i$ (d) $3-\sqrt{3}i$ (e) $-\frac{1}{\sqrt{3}}-i$

2. Find all the roots of the following equations and plot them on an Argand diagram:

   (a) $z^3 = -i$ (b) $z^3 = 8$ (c) $z^2 = i$ (d) $z^4 = 8\sqrt{2}(1-i)$

3. Use de Moivre's theorem $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, to find expressions for $\sin 3\theta$ and $\cos 3\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$.

4. Given that $\sqrt{3}-i = 2e^{11\pi/6}$, express the complex number $\left(\sqrt{3}-i\right)^{10}$ in standard form. ***Hint***: make use of the fact that adding integer multiples of 2π to the argument of a complex number leaves that complex number unchanged.

5. By writing $1-i$ in exponential form, show that

$$(1-i)^{99} = 2^{99/2}\left(-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right).$$

6. The cube roots of unity (i.e., the solutions to the equation $z^3 = 1$) are given by 1, $\exp(2\pi i/3)$ and $\exp(4\pi i/3)$. These three roots are often denoted by 1, ω and ω^2 respectively. Show that $\omega^3 = 1$ and that $1+\omega+\omega^2 = 0$.

7. Writing $z = \exp(i\theta)$, show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad \text{and} \qquad z^n - \frac{1}{z^n} = 2i\sin n\theta.$$

   Set $n = 1$ in the formula involving $\cos\theta$, and use this result to find an expression for $\cos^3\theta$ in terms of $\cos 3\theta$ and $\cos\theta$.

8. By considering the real and imaginary parts of the product $\exp(i\alpha)$ and $\exp(i\beta)$, show that

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

   and

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta.$$

	0	$\boldsymbol{\pi/6}$	$\boldsymbol{\pi/4}$	$\boldsymbol{\pi/3}$	$\boldsymbol{\pi/2}$
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	*