

1. You are given the function $f(x) = x \ln x$.
 - (a) State its domain, giving a brief explanation of your answer.
 - (b) Does the function intercept the x - and y -axes? If so, determine where.
 - (c) Determine whether the function is odd, even, or neither of these.
 - (d) Explain how we can tell that this function has no horizontal asymptotes. By calculating an appropriate limit, find whether $f(x)$ has a vertical asymptote as $x \rightarrow 0$.
 - (e) Determine the intervals of increase and decrease of the function.
 - (f) Find any local maxima and minima of the function, and give their x and y coordinates.
 - (g) Show that the function is concave up (i.e. it has $f'' > 0$) over the whole of its domain.
 - (h) Use your answers to parts (a) to (g) to sketch the graph of $f(x)$.

2. For the function $g(x) = \frac{2(x-2)}{x^2}$, answer the following questions:
 - (a) State the domain of this function, giving a brief explanation of your answer.
 - (b) Does $g(x)$ intercept the x - and y -axes? If so, determine where.
 - (c) Determine whether the function is odd, even, or neither of these.
 - (d) Find the vertical and horizontal asymptotes of this function.
 - (e) Find the intervals of increase and decrease of $g(x)$.
 - (f) Find any local maxima and minima of the function, and give their x and y coordinates.
 - (g) Find where, if anywhere, the function is concave up and concave down. Determine whether it has any points of inflection, and, if so, give the x and y coordinates of these.
 - (h) Use your answers to parts (a) to (g) to sketch the graph of $g(x)$.

3. For $h(x) = \frac{x^2}{\sqrt{x+1}}$, answer the following:
 - (a) State the domain of $h(x)$, giving a short explanation.
 - (b) Find where $h(x)$ intercepts the x - and y -axes.
 - (c) Determine whether the function is odd, even, or neither of these.
 - (d) (i) Show that $h(x)$ has no horizontal asymptote.
(ii) Locate the vertical asymptote of the function.
 - (e) Determine on which intervals $h(x)$ increases and decreases.
 - (f) Find the local extremum of the function, and classify it as a maximum or minimum.
 - (g) Show that $h(x)$ is concave up over the whole of its domain.
 - (h) Use your answers to the preceding parts of the question to sketch the graph of $h(x)$.