Wave Optics

Wave Optics

Wave optics is a study concerned with phenomena that cannot be adequately explained by geometric (ray) optics.

These phenomena include:

- Interference
- Diffraction

Interference

In *constructive interference* the amplitude of the resultant wave is greater than that of either individual wave.

In *destructive interference* the amplitude of the resultant wave is less than that of either individual wave.

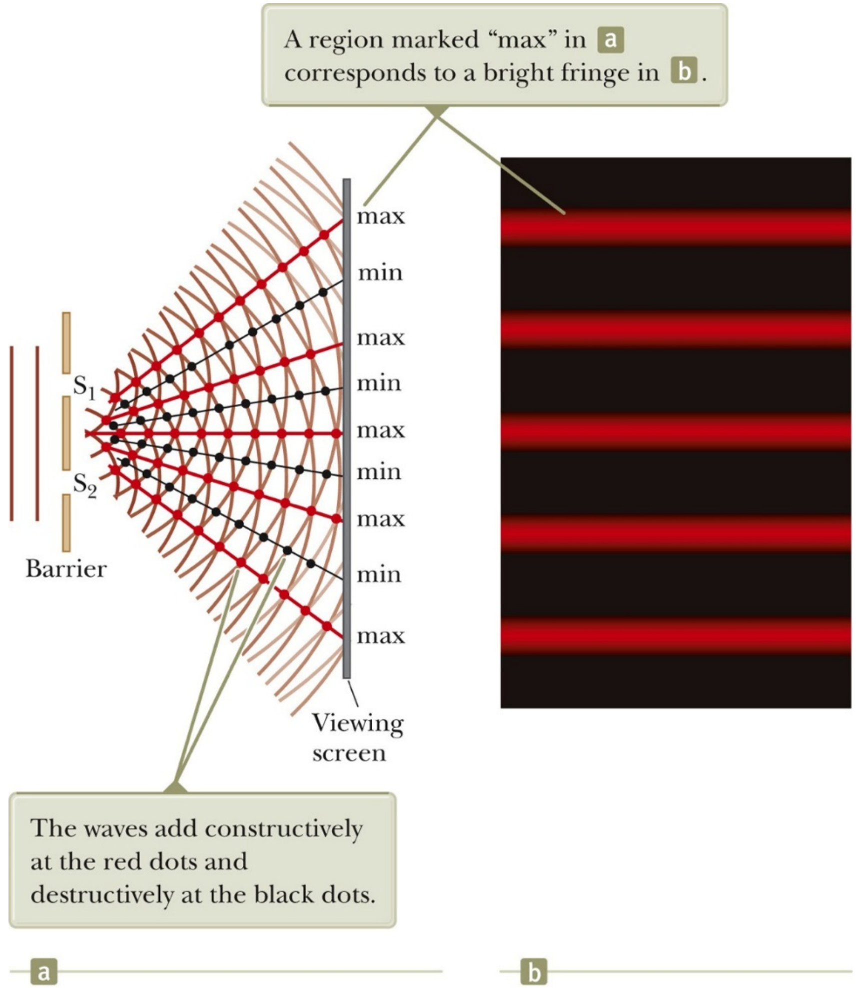
All interference associated with light waves arises when the electromagnetic fields that constitute the individual waves combine.

Young’s Double-Slit Experiment: Schematic

Thomas Young first demonstrated interference in light waves from two sources in 1801.

The narrow slits S_1 and S_2 act as sources of waves.

The waves emerging from the slits originate from the same wave front and therefore are always in phase.



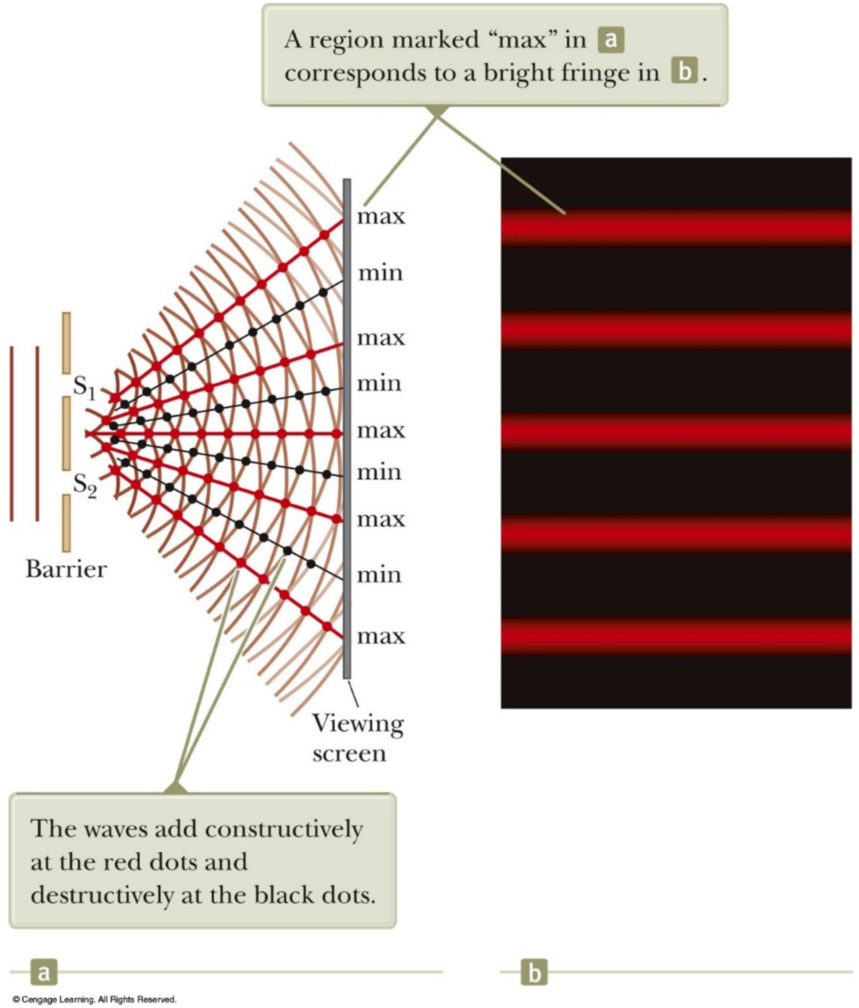
Resulting Interference Pattern

The light from the two slits forms a visible pattern on a screen.

The pattern consists of a series of bright and dark parallel bands called *fringes.*

Constructive interference occurs where a bright fringe occurs.

Destructive interference results in a dark fringe.



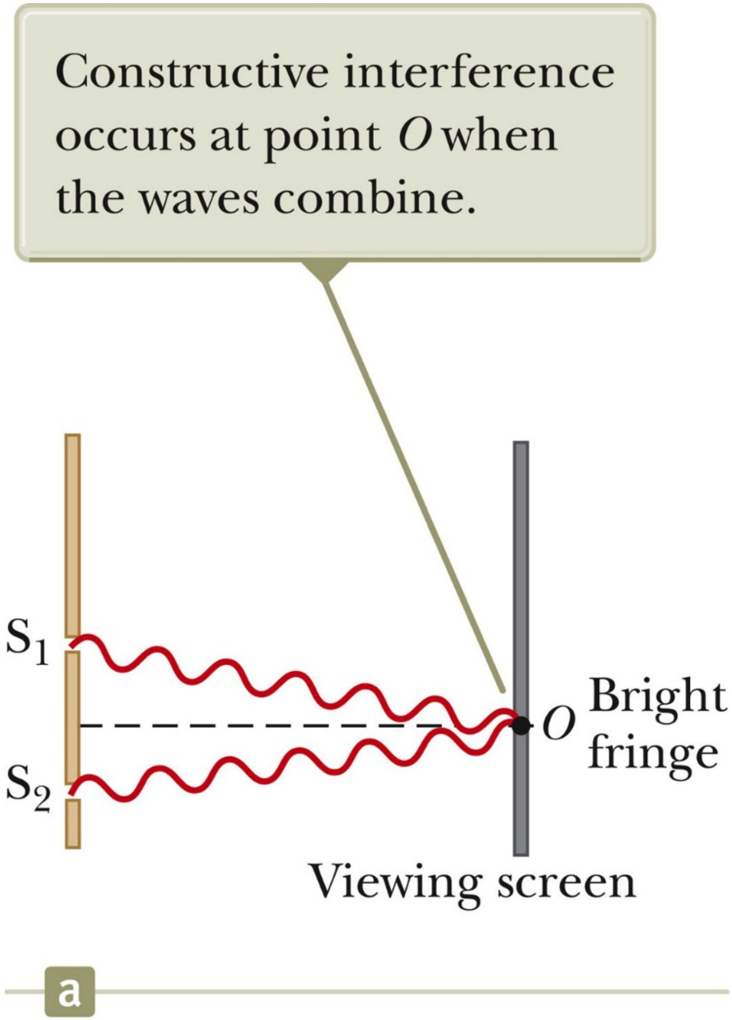
Interference Patterns

Constructive interference occurs at point O.

The two waves travel the same distance.

- Therefore, they arrive in phase

As a result, constructive interference occurs at this point and a bright fringe is observed.



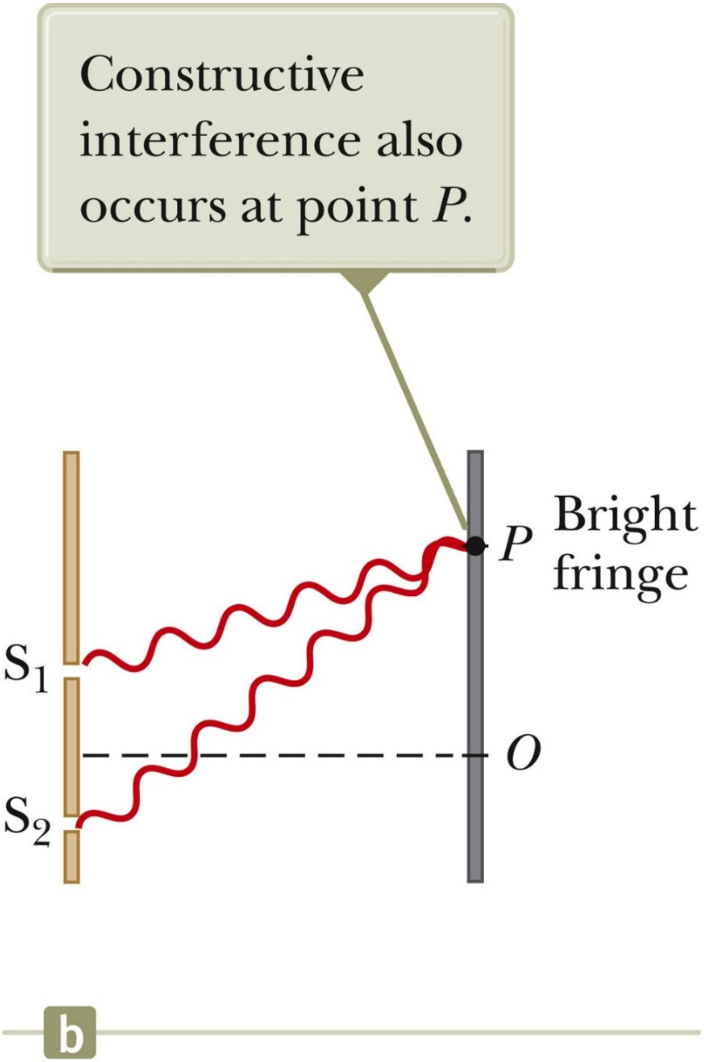
Interference Patterns, 2

The lower wave has to travel farther than the upper wave to reach point *P.*

The lower wave travels one wavelength farther.

- Therefore, the waves arrive in phase

A second bright fringe occurs at this position.



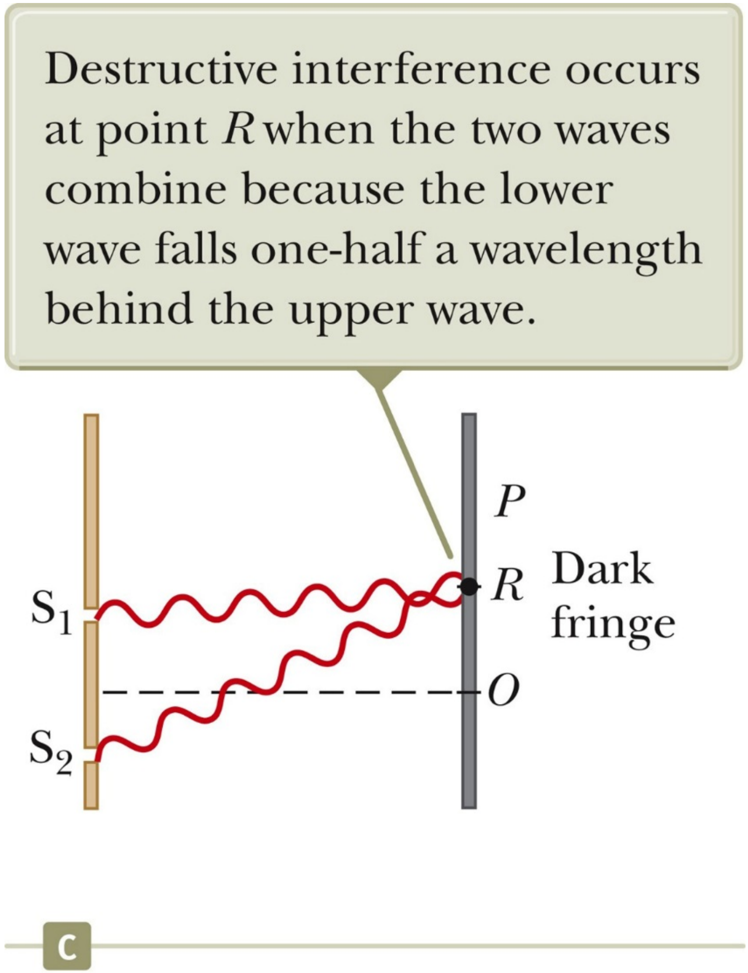
Interference Patterns, 3

The upper wave travels one-half of a wavelength farther than the lower wave to reach point *R.*

The trough of the upper wave overlaps the crest of the lower wave.

This is destructive interference.

- A dark fringe occurs.



Conditions for Interference

To observe interference in light waves, the following two conditions must be met:

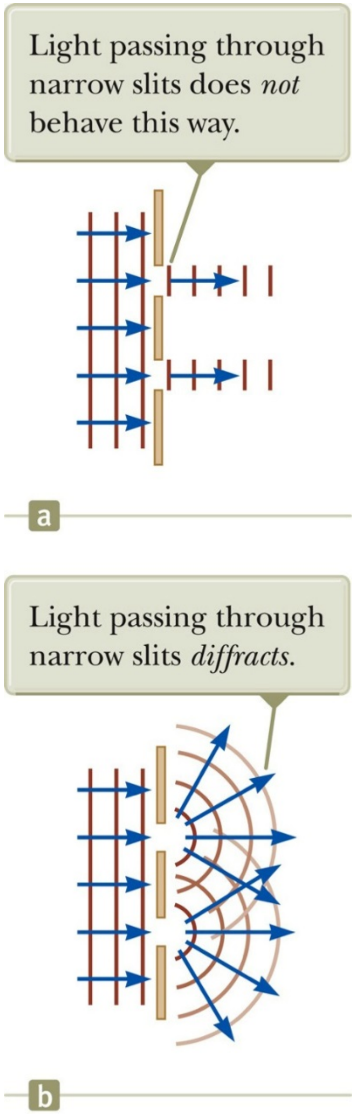
- The sources must be **coherent.**
  - They must maintain a constant phase with respect to each other.
- The sources should be **monochromatic.**
  - Monochromatic means they have a single wavelength.

Diffraction

If the light traveled in a straight line after passing through the slits, no interference pattern would be observed.

From Huygens's principle we know the waves spread out from the slits.

This divergence of light from its initial line of travel is called **diffraction.**

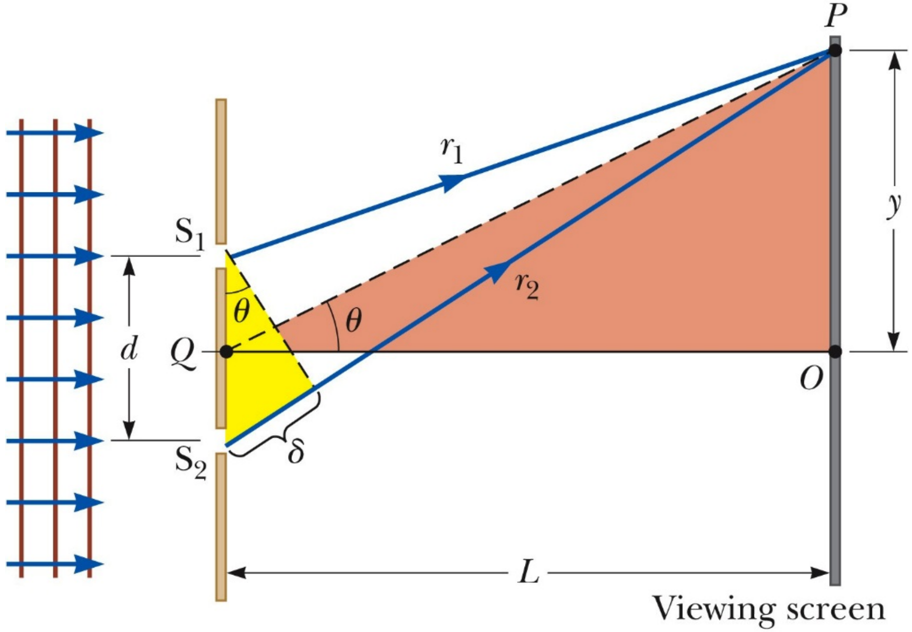


Young's Double-Slit Experiment: Geometry

The path difference, δ, is found from geometry.

$\delta = r_2 - r_1 = d \sin \theta$

- This assumes the paths are parallel.
- Not exactly true, but a very good approximation if *L* is much greater than *d*



Interference Equations

For a bright fringe produced by constructive interference, the path difference must be either zero or some integer multiple of the wavelength.

$\delta = d \sin \theta_{bright} = m\lambda$

- $m = 0, \pm 1, \pm 2, \ldots$
- m is called the order number
  - When $m = 0$, it is the zeroth-order maximum
  - When $m = \pm 1$, it is called the first-order maximum

When destructive interference occurs, a dark fringe is observed.

This needs a path difference of an odd half wavelength.

$\delta = d \sin \theta_{dark} = (m + \frac{1}{2})\lambda$

- $m = 0, \pm 1, \pm 2, \ldots$

Interference Equations, cont.

The positions of the fringes can be measured vertically from the zeroth-order maximum.

Using the large triangle in figure,

- $y_{bright} = L \tan \theta_{bright}$
- $y_{dark} = L \tan \theta_{dark}$

Interference Equations, final

Assumptions in a Young's Double Slit Experiment:

- L >> d
- d >> λ

Approximation:

- θ is small and therefore the small angle approximation tan θ ~ sin θ can be used

y = L tan θ ≈ L sin θ

For small angles,

$$y_{\text{bright}} = L\frac{m\lambda}{d} \text{ and } y_{\text{dark}} = L\frac{(m+\frac{1}{2})\lambda}{d}$$

Uses for Young's Double-Slit Experiment

Young's double-slit experiment provides a method for measuring wavelength of the light.

This experiment gave the wave model of light a great deal of credibility.

- It was inconceivable that particles of light could cancel each other in a way that would explain the dark fringes.

Multiple Slits, Graph

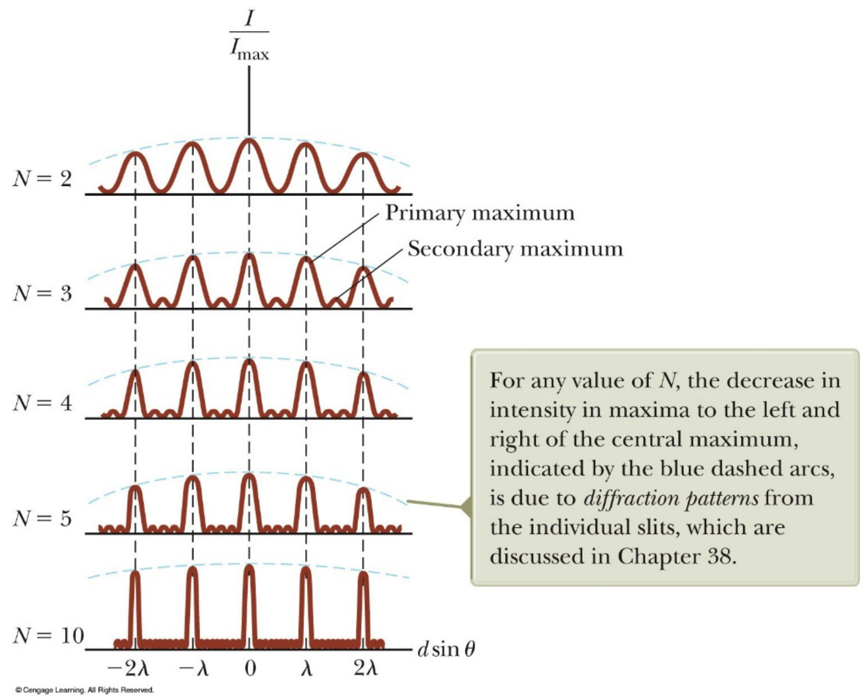
With more than two slits, the pattern contains primary and secondary maxima.

For N slits, the intensity of the primary maxima is N^2 times greater than that due to a single slit.

As the number of slits increases, the primary maxima increase in intensity and become narrower.

- The secondary maxima decrease in intensity relative to the primary maxima.

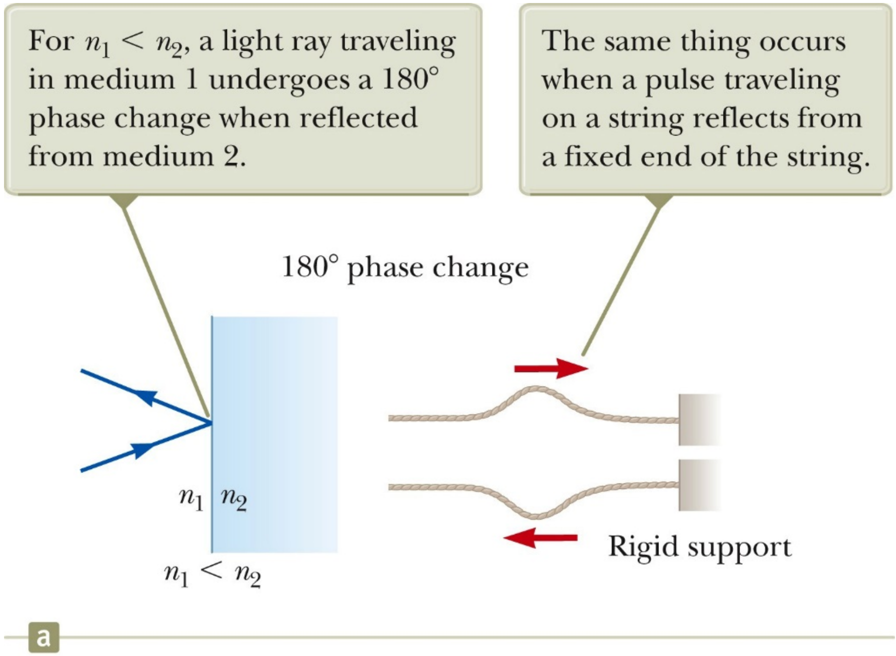
The number of secondary maxima is N – 2, where N is the number of slits.



Phase Changes Due To Reflection

An electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was traveling.

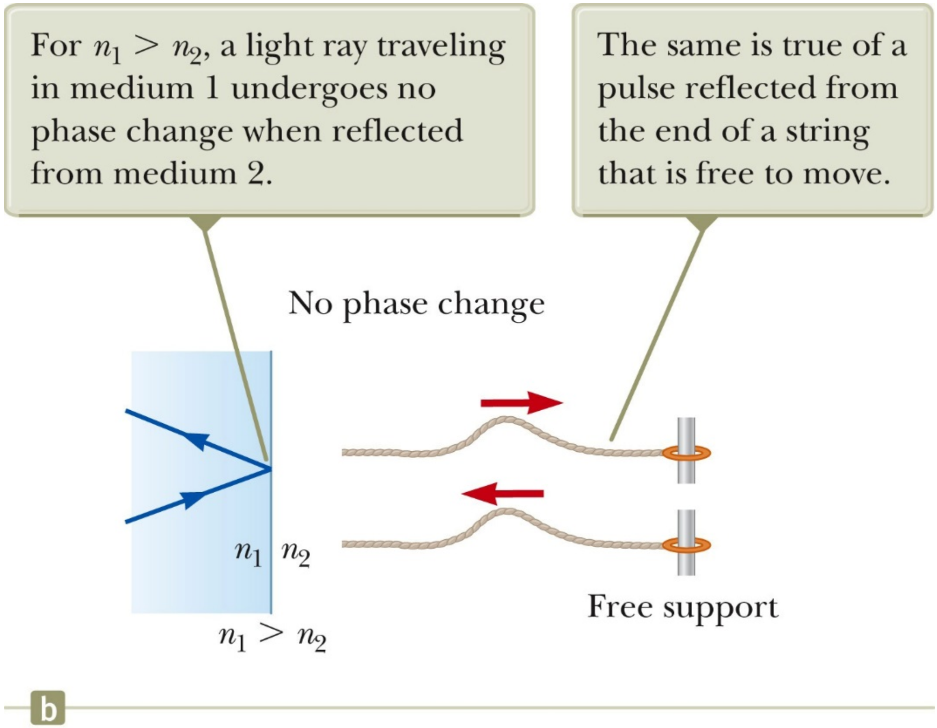
- Analogous to a pulse on a string reflected from a rigid support



Phase Changes Due To Reflection, cont.

There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction.

- Analogous to a pulse on a string reflecting from a free support



Interference in Thin Films

Interference effects are commonly observed in thin films.

- Examples include soap bubbles and oil on water

The various colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

Facts to remember:

- An electromagnetic wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change on reflection when $n_2 > n_1$.
  - There is no phase change in the reflected wave if $n_2 < n_1$.
- The wavelength of light λ_n in a medium with index of refraction n is $\lambda_n = \lambda/n$ where λ is the wavelength of light in vacuum.

Interference in Thin Films, 2

Assume the light rays are traveling in air nearly normal to the two surfaces of the film.

Ray 1 undergoes a phase change of 180° with respect to the incident ray.

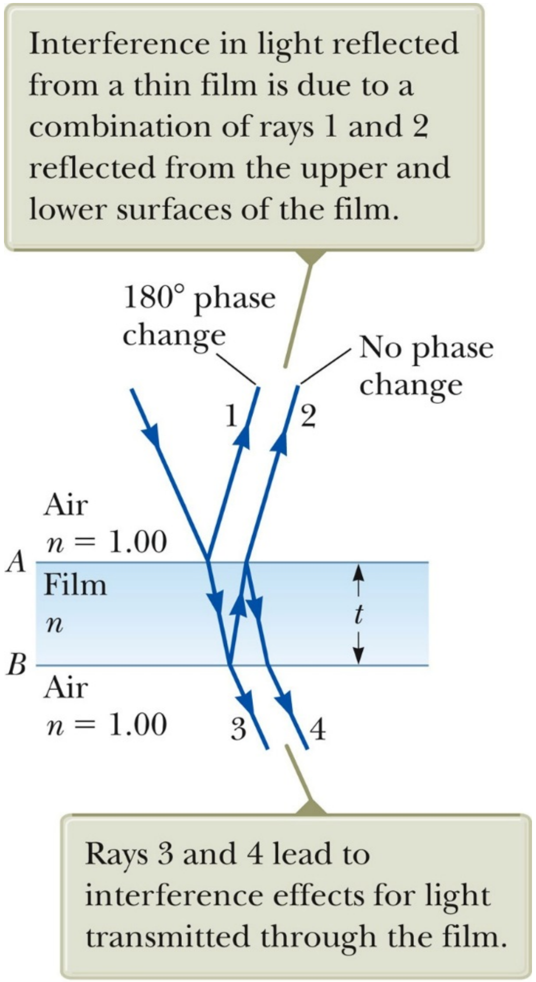
Ray 2, which is reflected from the lower surface, undergoes no phase change with respect to the incident wave. Ray 2 also travels an additional distance of 2t before the waves recombine.

For constructive interference

- $2nt = (m + \frac{1}{2})\lambda \qquad (m = 0, 1, 2 \ldots)$
  - This takes into account both the difference in optical path length for the two rays and the 180° phase change.

For destructive interference

- $2nt = m\lambda \qquad (m = 0, 1, 2 \ldots)$



Interference in Thin Film, Soap Bubble Example

Dr. Jeremy Burgess/Science Photo Library/Photo Researchers, Inc.

Newton’s Rings

Another method for viewing interference is to place a plano-convex lens on top of a flat glass surface.

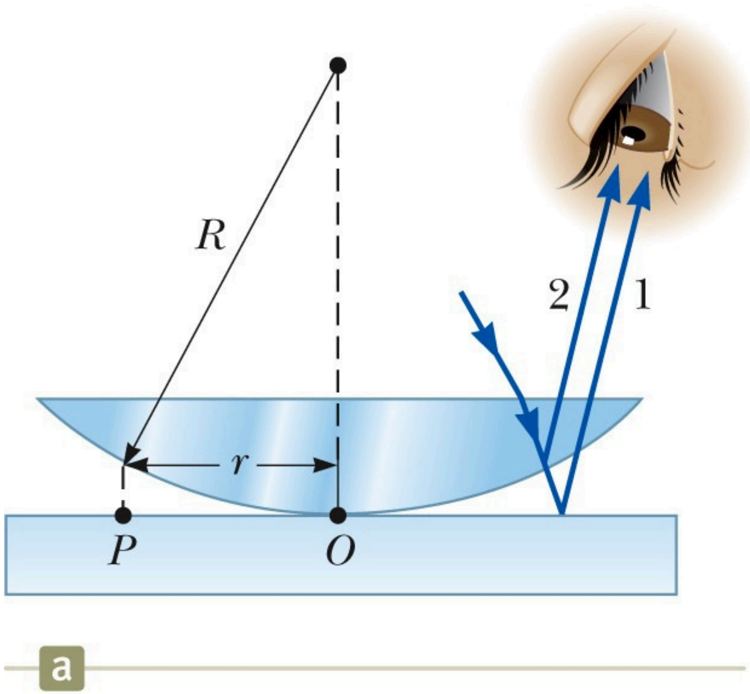
The air film between the glass surfaces varies in thickness from zero at the point of contact to some thickness t.

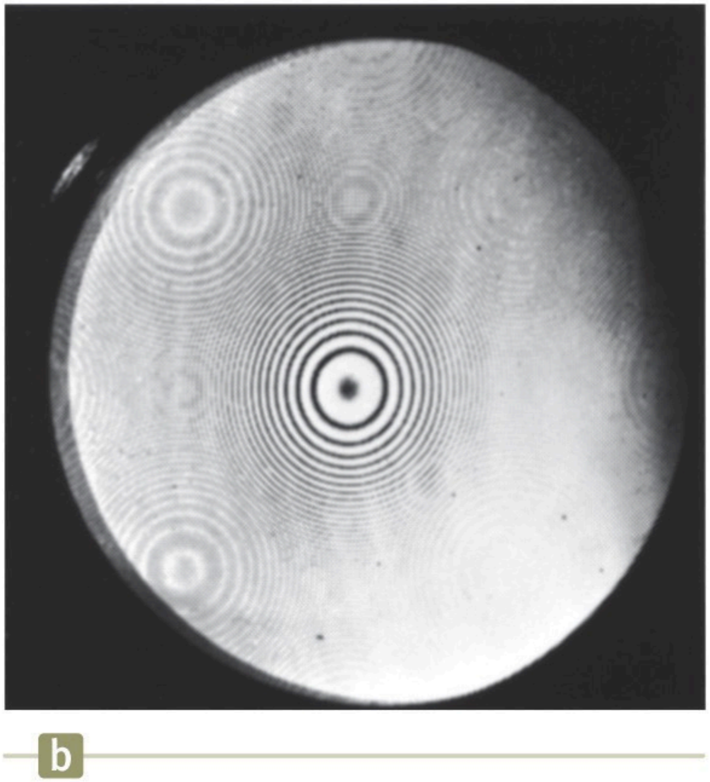
A pattern of light and dark rings is observed.

- These rings are called Newton’s rings.
- The particle model of light could not explain the origin of the rings.

Newton’s rings can be used to test optical lenses.

Newton's Rings, Set-Up and Pattern





Diffraction Pattern

A single slit placed between a distant light source and a screen produces a **diffraction pattern.**

- It will have a broad, intense central band
  - Called the **central maximum**
- The central band will be flanked by a series of narrower, less intense secondary bands.
  - Called **side maxima** or **secondary maxima**
- The central band will also be flanked by a series of dark bands.
  - Called **minima**

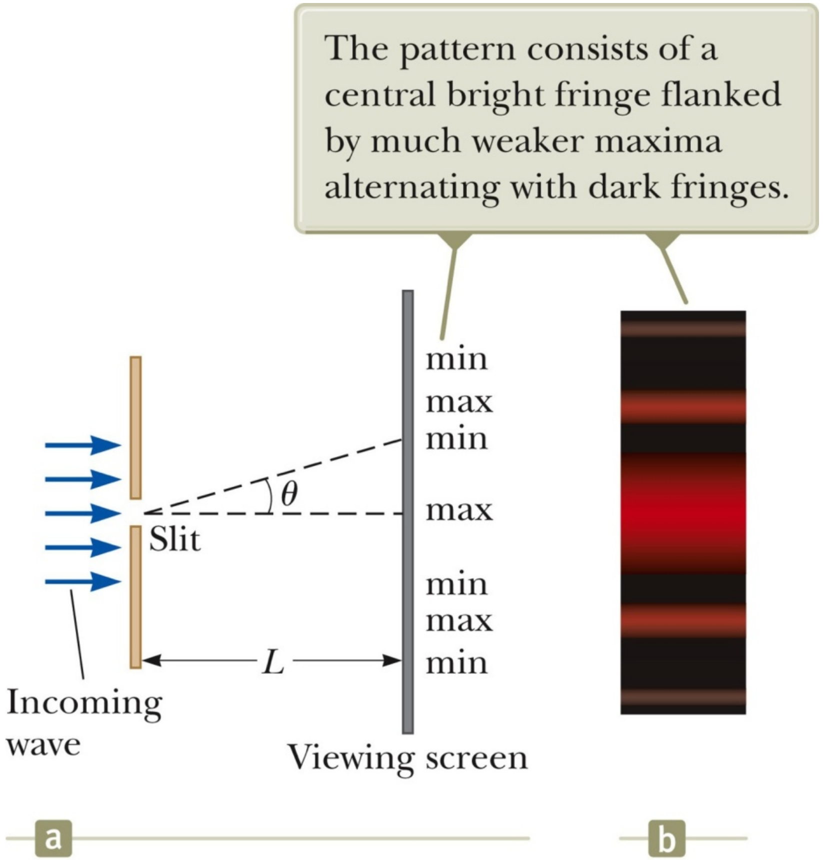
Fraunhofer Diffraction Pattern

A **Fraunhofer diffraction pattern** occurs when the rays leave the diffracting object in parallel directions.

- Screen very far from the slit

A bright fringe is seen along the axis ($\theta = 0$).

Alternating bright and dark fringes are seen on each side.



Single-Slit Diffraction

The finite width of slits is the basis for understanding Fraunhofer diffraction.

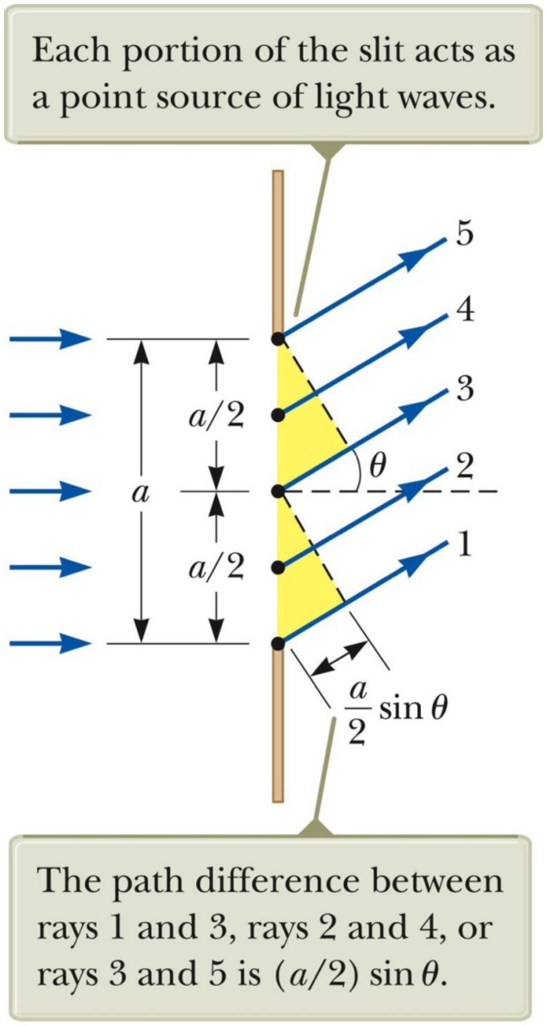
According to Huygens's principle, each portion of the slit acts as a source of light waves.

Therefore, light from one portion of the slit can interfere with light from another portion.

The resultant light intensity on a viewing screen depends on the direction *θ.*

The diffraction pattern is actually an interference pattern.

- The different sources of light are different portions of the single slit.



Single-Slit Diffraction, Analysis

All the waves are in phase as they leave the slit.

Wave 1 travels farther than wave 3 by an amount equal to the path difference.

- $(a/2) \sin \theta$

If this path difference is exactly half of a wavelength, the two waves cancel each other and destructive interference results.

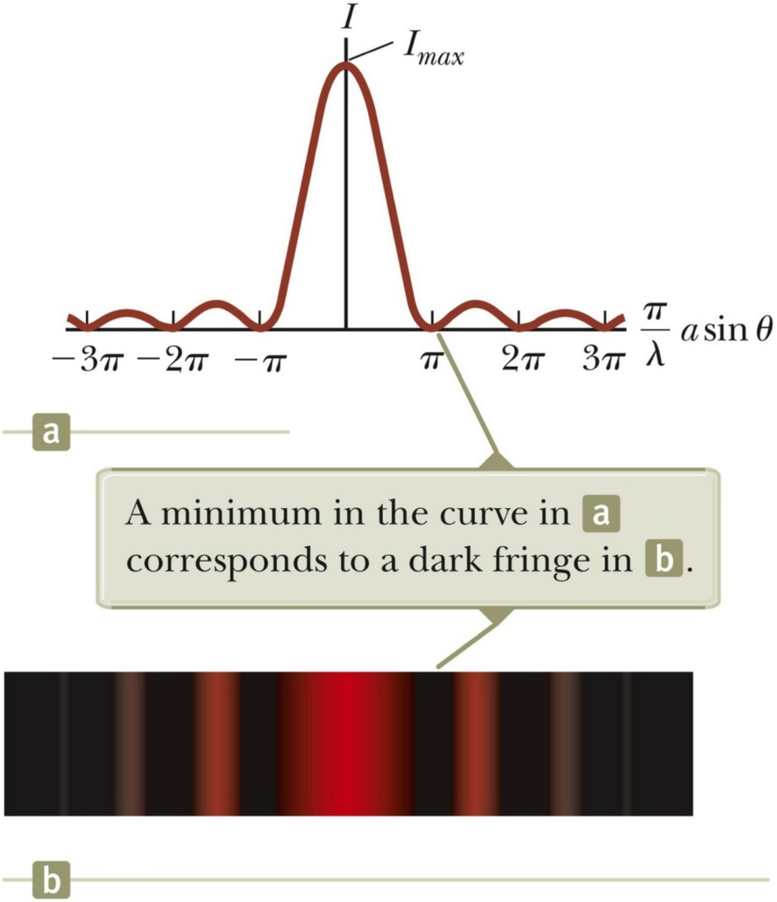
In general, destructive interference occurs for a single slit of width a when $\sin \theta_{dark} = m\lambda / a$.

- $m = \pm 1, \pm 2, \pm 3, \ldots$

Intensity

Most of the light intensity is concentrated in the central maximum.

The graph shows a plot of light intensity vs. (π / λ) a sin θ.

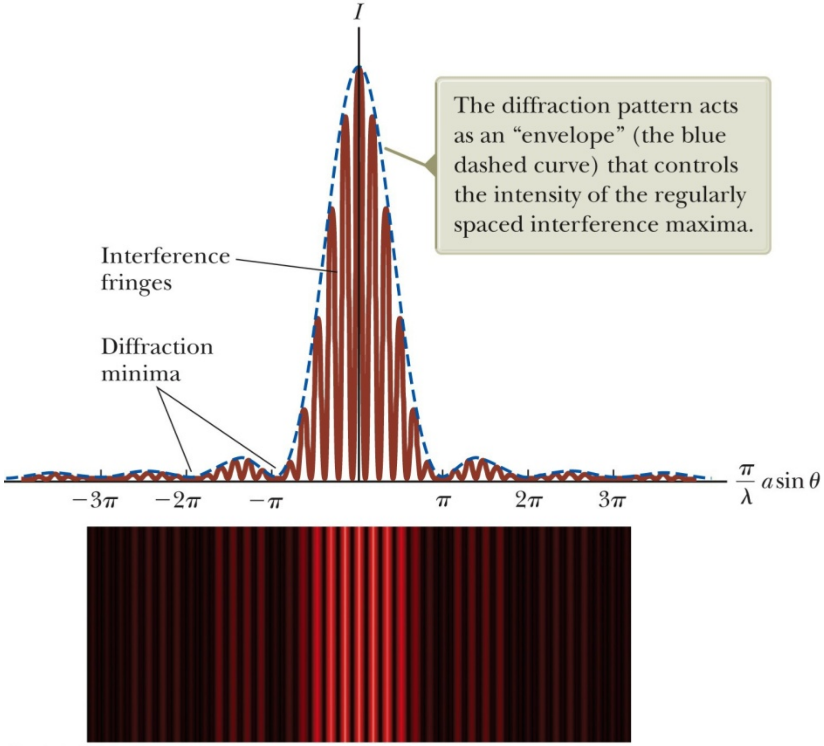


Intensity of Two-Slit Diffraction Patterns

When more than one slit is present, consideration must be made of

- The diffraction patterns due to individual slits
- The interference due to the wave coming from different slits

The single-slit diffraction pattern will act as an “envelope” for a two-slit interference pattern.



Two-Slit Diffraction Patterns, Maxima and Minima

To find which interference maximum coincides with the first diffraction minimum.

$$\frac{d \sin \theta}{a \sin \theta} = \frac{m\lambda}{\lambda} \rightarrow \frac{d}{a} = m$$

- The conditions for the mth interference maximum
  - d sin θ = m λ
- The conditions for the first diffraction minimum
  - a sin θ = λ

Resolution

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light.

If two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished.

- The images are said to be resolved.

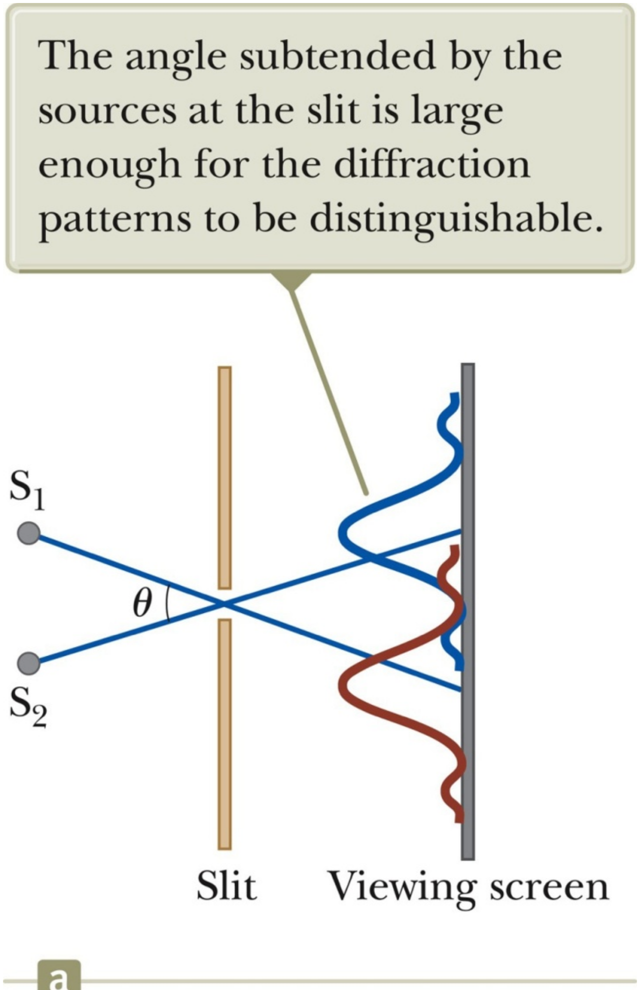
If the two sources are close together, the two central maxima overlap and the images are not resolved.

Resolved Images, Example

The images are far enough apart to keep their central maxima from overlapping.

The angle subtended by the sources at the slit is large enough for the diffraction patterns to be distinguishable.

The images are resolved.

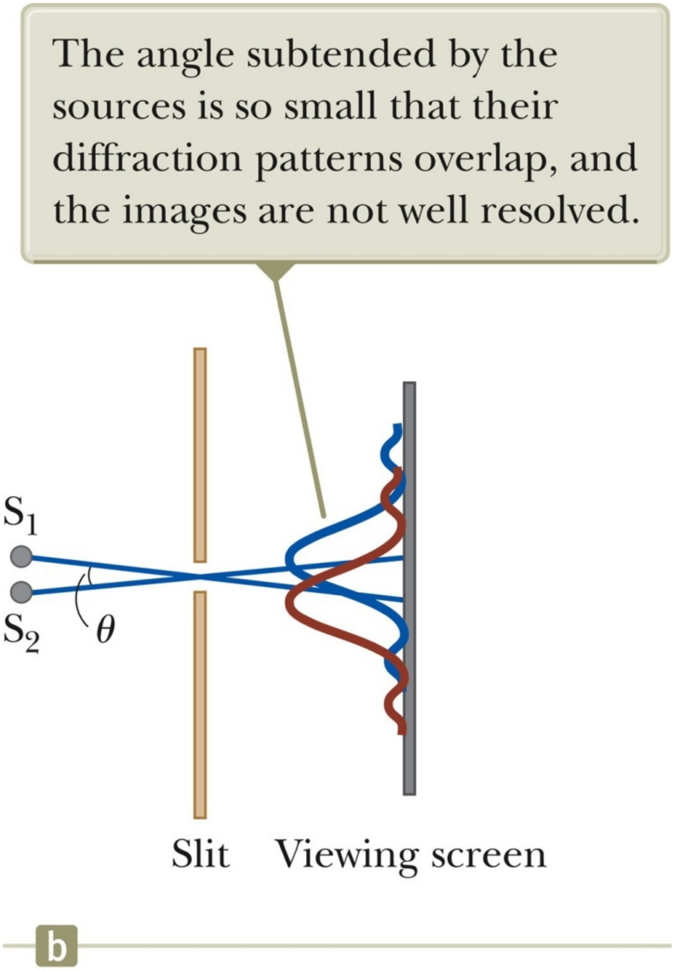


Images Not Resolved, Example

The sources are so close together that their central maxima do overlap.

The angle subtended by the sources is so small that their diffraction patterns overlap.

The images are not resolved.



Resolution, Rayleigh’s Criterion

When the central maximum of one image falls on the first minimum of another image, the images are said to be just resolved.

This limiting condition of resolution is called **Rayleigh’s criterion.**

The angle of separation, θ_{min}, is the angle subtended by the sources for which the images are just resolved.

Since $\lambda << a$ in most situations, $\sin \theta$ is very small and $\sin \theta \approx \theta$.

Therefore, the limiting angle (in rad) of resolution for a slit of width a is

$$\theta_{\mathrm{min}} = \frac{\lambda}{a}$$

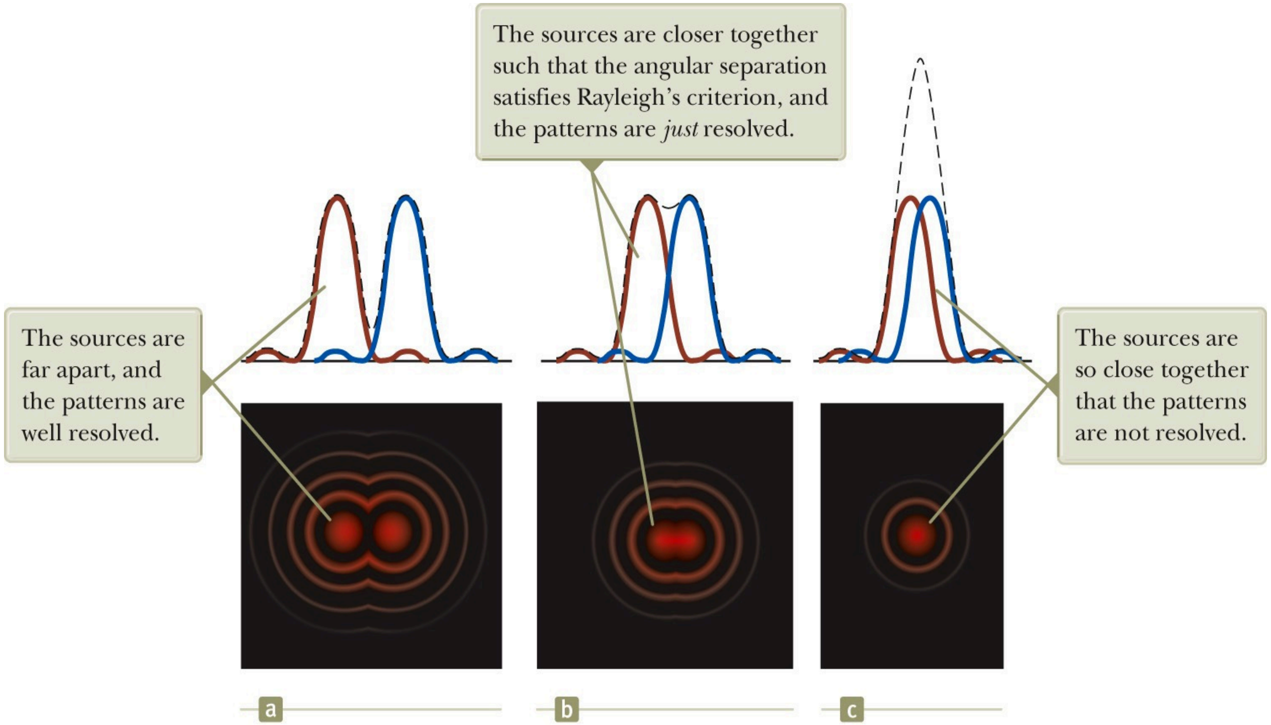
Circular Apertures

The diffraction pattern of a circular aperture consists of a central bright disk surrounded by progressively fainter bright and dark rings.

The limiting angle of resolution of the circular aperture is

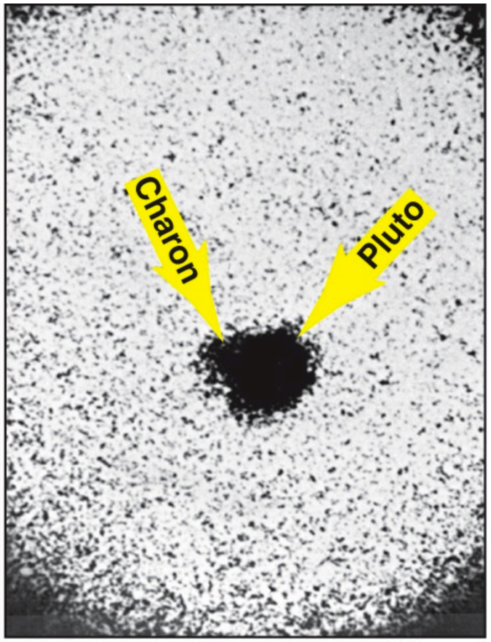
$$\theta_{\min} = 1.22\,\frac{\lambda}{D}$$

- *D* is the diameter of the aperture.



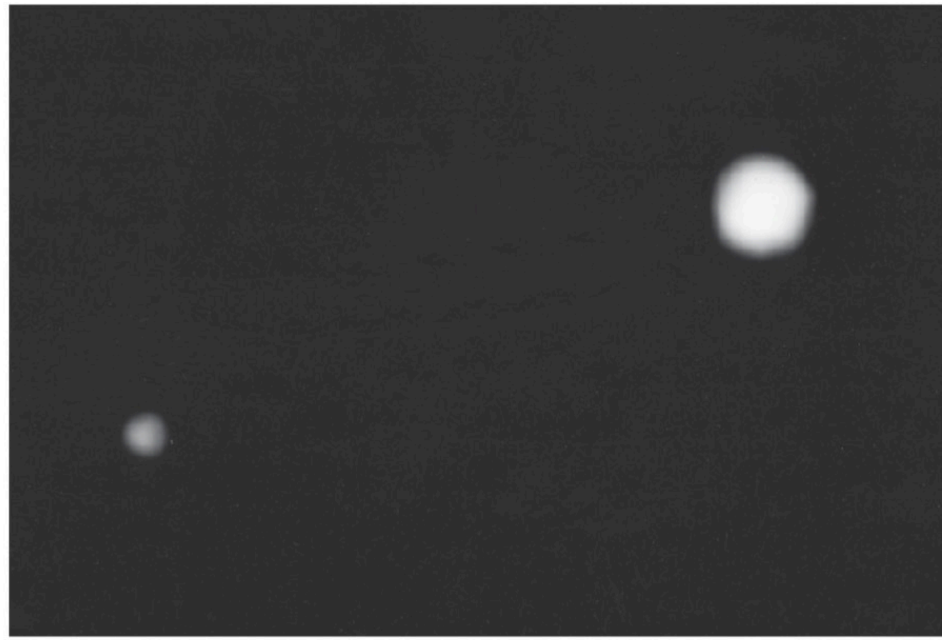
Resolution, Example

Courtesy U.S. Naval Observatory/James W. Christy



a

Dr. R. Albrecht, ESA/ESO Space Telescope European Coordinating Facility; NASA



b

Pluto and its moon, Charon

Left: Earth-based telescope is blurred

Right: Hubble Space Telescope clearly resolves the two objects

Diffraction Grating

The diffracting grating consists of a large number of equally spaced parallel slits.

- A typical grating contains several thousand lines per centimeter.

The intensity of the pattern on the screen is the result of the combined effects of interference and diffraction.

- Each slit produces diffraction, and the diffracted beams interfere with one another to form the final pattern.

Diffraction Grating, Types

A *transmission* grating can be made by cutting parallel grooves on a glass plate.

- The spaces between the grooves are transparent to the light and so act as separate slits.

A *reflection* grating can be made by cutting parallel grooves on the surface of a reflective material.

- The spaces between the grooves act as parallel sources of reflected light, like the slits in a transmission grating.

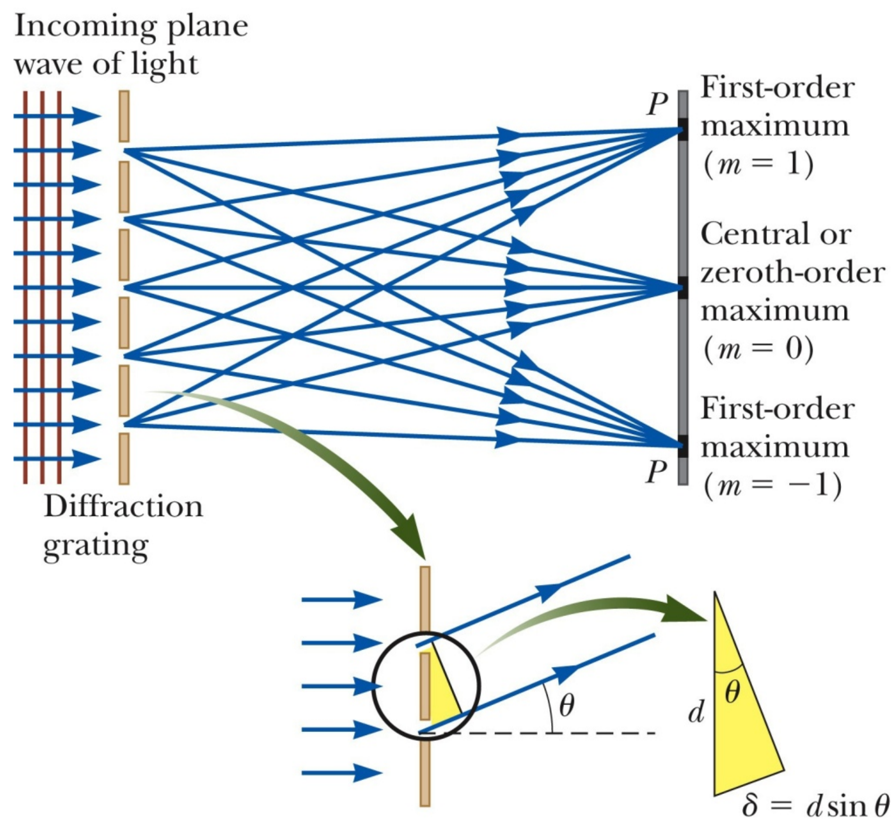
Diffraction Grating, cont.

The condition for *maxima* is

- $d \sin \theta_{bright} = m\lambda$
  - $m = 0, \pm 1, \pm 2, \ldots$

The integer m is the *order number* of the diffraction pattern.

If the incident radiation contains several wavelengths, each wavelength deviates through a specific angle.



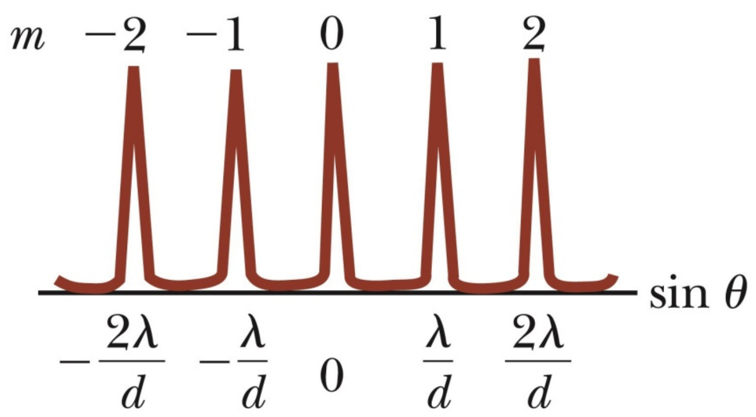
Diffraction Grating, Intensity

All the wavelengths are seen at *m* = 0.

- This is called the zeroth-order maximum.

The first-order maximum corresponds to *m* = 1.

Note the sharpness of the principle maxima and the broad range of the dark areas.



Characteristics of the intensity pattern:

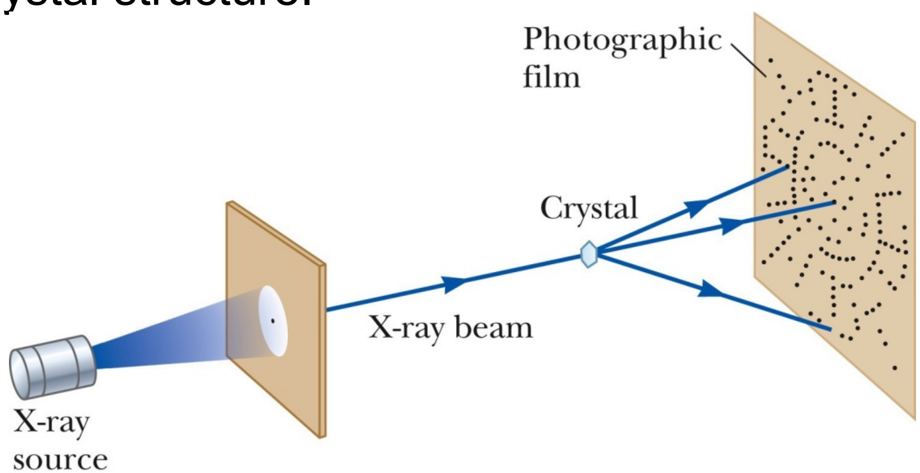
- The sharp peaks are in contrast to the broad, bright fringes characteristic of the two-slit interference pattern.
- Because the principle maxima are so sharp, they are much brighter than two-slit interference patterns.

Diffraction of X-Rays by Crystals

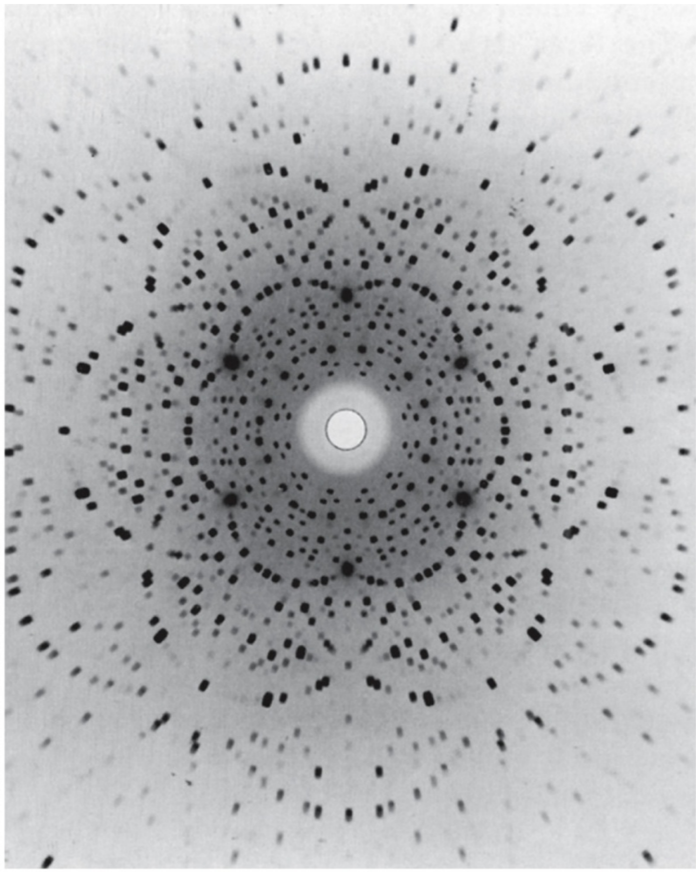
X-rays are electromagnetic waves of very short wavelength.

Max von Laue suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for x-rays.

The diffraction patterns from crystals are complex because of the three-dimensional nature of the crystal structure.



Laue Pattern for Beryl



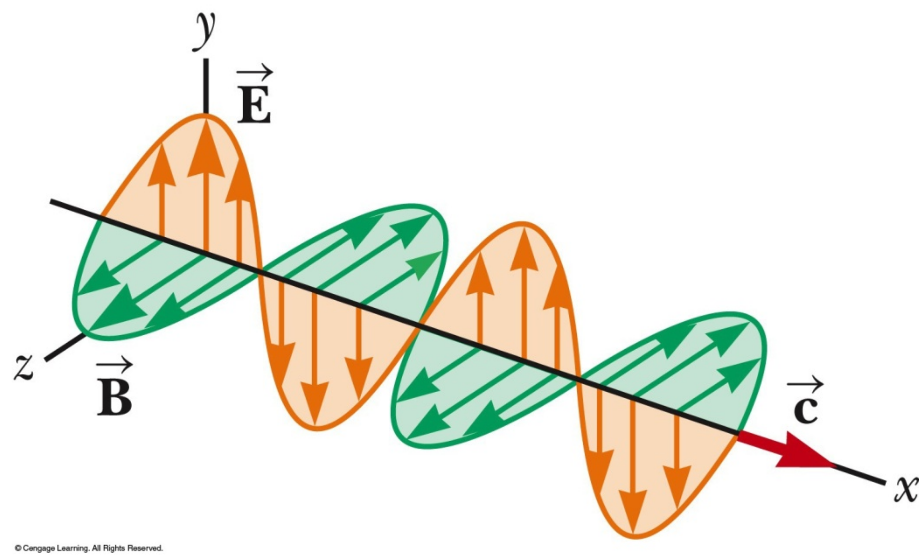
Polarization of Light Waves

The *direction of polarization* of each individual wave is defined to be the direction in which the electric field is vibrating.

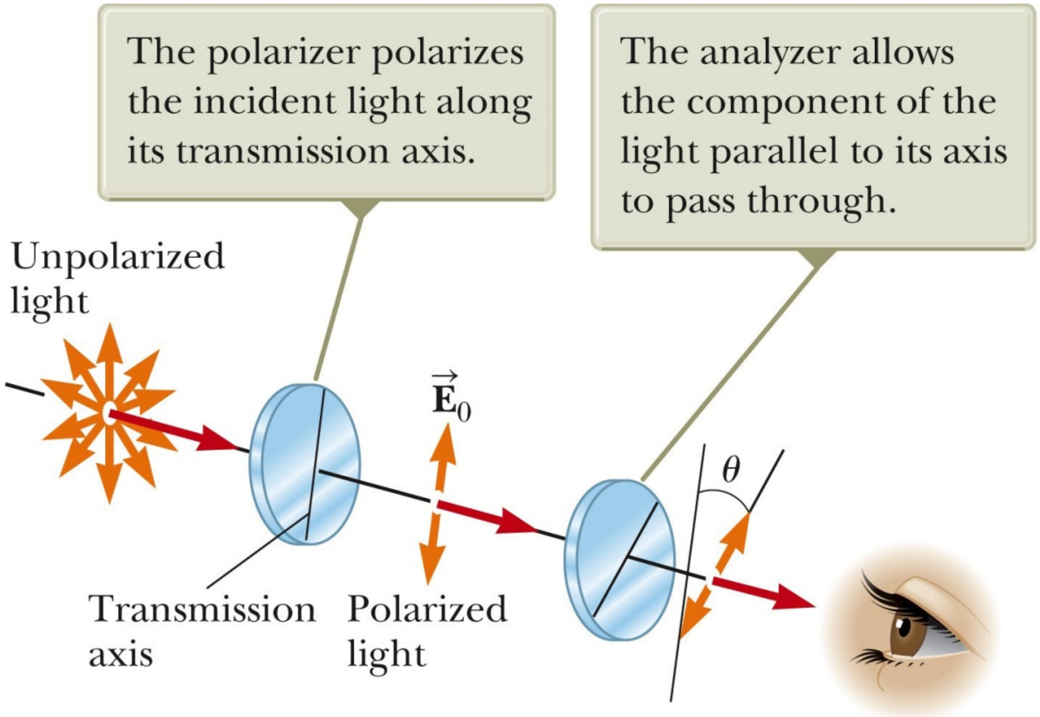
In this example, the direction of polarization is along the *y*-axis.

All individual electromagnetic waves traveling in the *x* direction have an electric field vector parallel to the *yz* plane.

This vector could be at any possible angle with respect to the *y* axis.



Polarization by Selective Absorption



The most common technique for polarizing light.

Uses a material that transmits waves whose electric field vectors lie in the plane parallel to a certain direction and absorbs waves whose electric field vectors are in all other directions.

Selective Absorption, cont.

E. H. Land discovered a material that polarizes light through selective absorption.

- He called the material *Polaroid.*
- The molecules readily absorb light whose electric field vector is parallel to their lengths and allow light through whose electric field vector is perpendicular to their lengths.

It is common to refer to the direction perpendicular to the molecular chains as the *transmission axis.*

In an *ideal* polarizer,

- All light with the electric field parallel to the transmission axis is transmitted.
- All light with the electric field perpendicular to the transmission axis is absorbed.

Intensity of a Polarized Beam

The intensity of the polarized beam transmitted through the second polarizing sheet (the analyzer) varies as

- $I = I_{max} \cos^2 \theta$
  - I_{max} is the intensity of the polarized wave incident on the analyzer.
  - This is known as **Malus' law** and applies to any two polarizing materials whose transmission axes are at an angle of θ to each other.

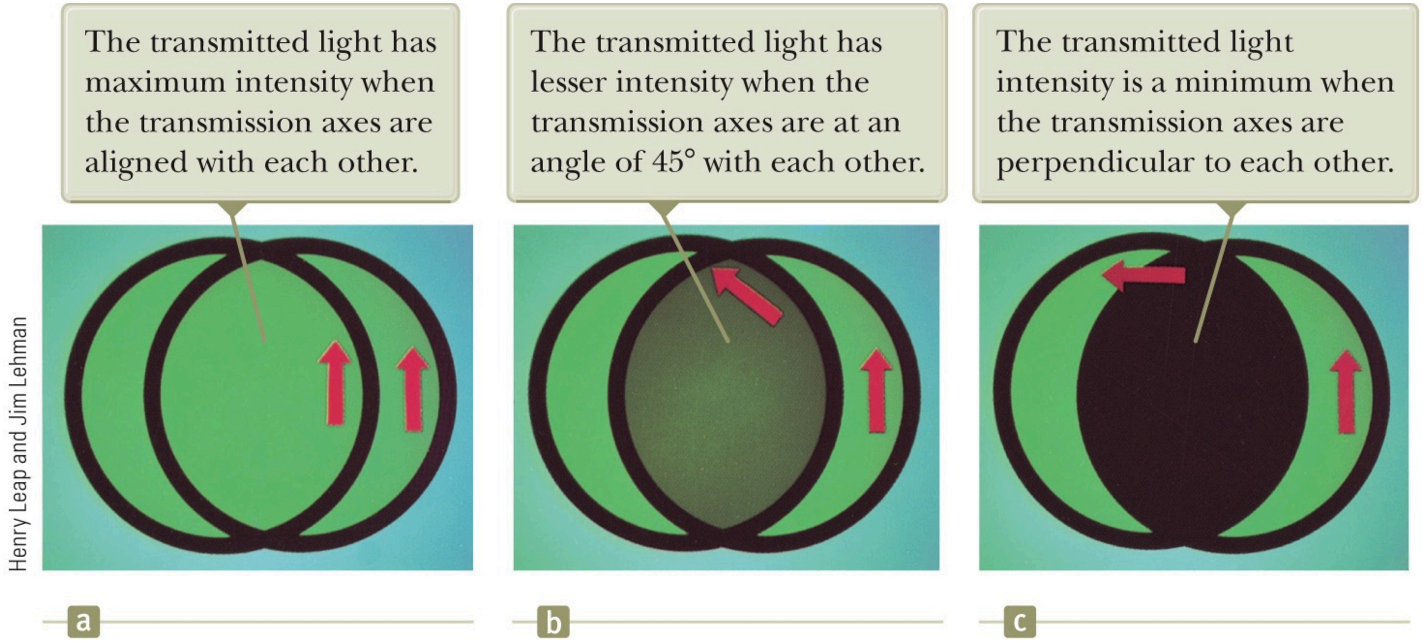
The intensity of the transmitted beam is a maximum when the transmission axes are parallel.

- $\theta = 0$ or 180^o

The intensity is zero when the transmission axes are perpendicular to each other.

- This would cause complete absorption.

Intensity of Polarized Light, Examples



On the left, the transmission axes are aligned and maximum intensity occurs.

In the middle, the axes are at 45° to each other and less intensity occurs.

On the right, the transmission axes are perpendicular and the light intensity is a minimum.