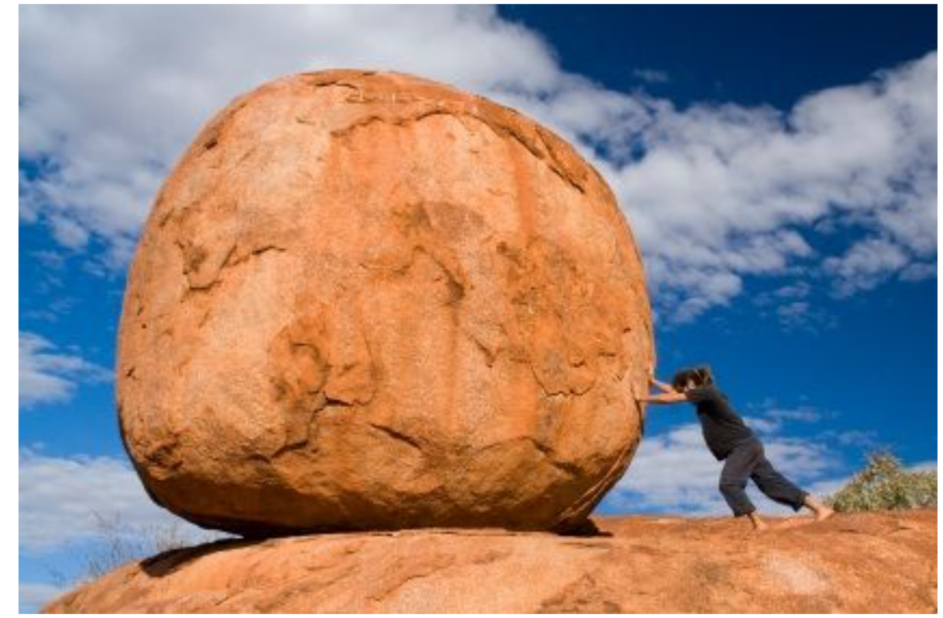
Friction forces

Solid friction force

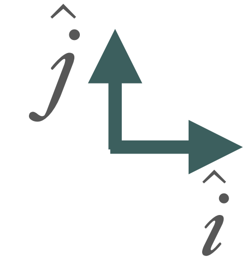
Solid friction

Can we make a 1000 tonnes boulder move by simply applying a reasonable force?



Let's try to figure it out!!

Solid friction



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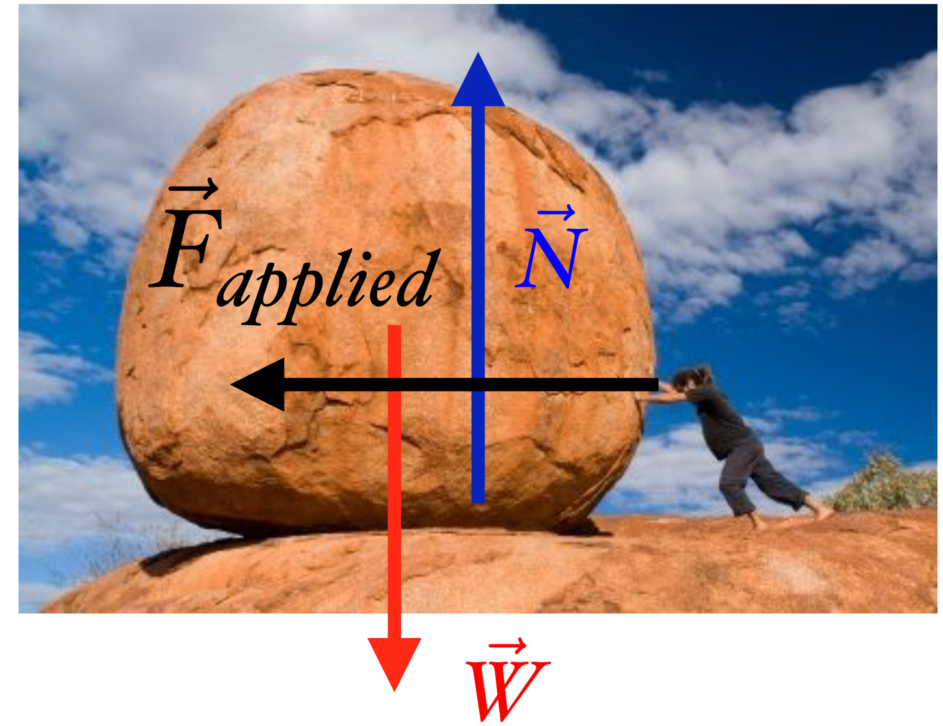
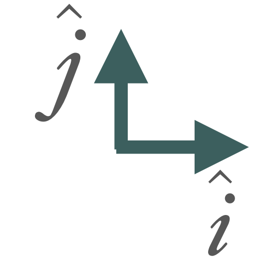


Let's try to figure it out!!

1st step: make a diagram with all the forces acting on the boulder

Solid friction



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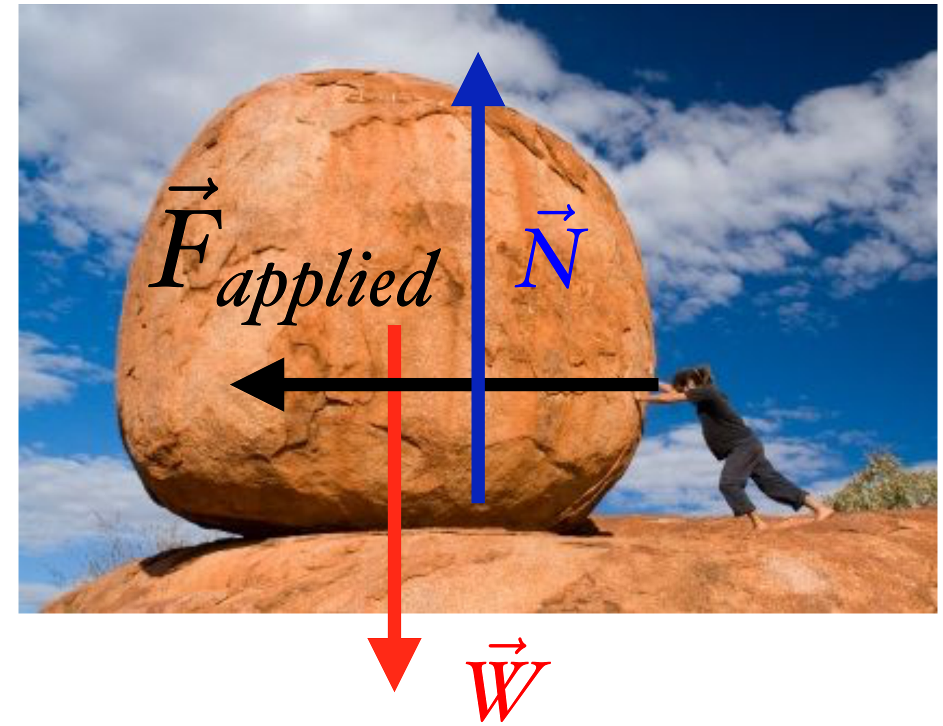
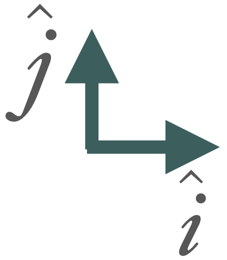


2nd step: list the forces acting on the boulder

Contact forces	**Forces at a distance**
$\vec{F}_{applied} = F_{applied}\,\hat{i}$	$\vec{W} = -m\,g\,\hat{j}, \quad g = 9.8\ \mathrm{m \cdot s^{-2}}$
$\vec{N} = N\hat{j}$	

Solid friction



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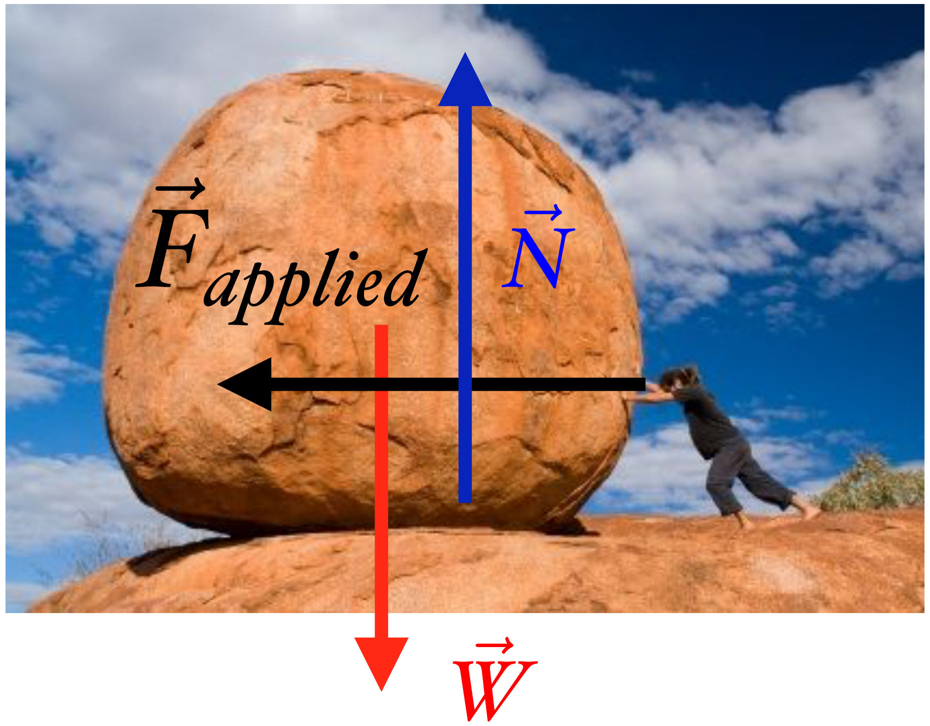


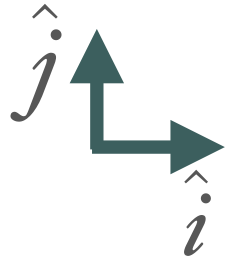
Let's try to figure it out!!

3rd step: apply Newton's 2nd law

We assume the frame of reference to be Galilean and therefore we can apply Newton's 2nd law

$$m\,\vec{a} = \vec{F}_{applied} + \vec{N} + \vec{W}$$

Solid friction



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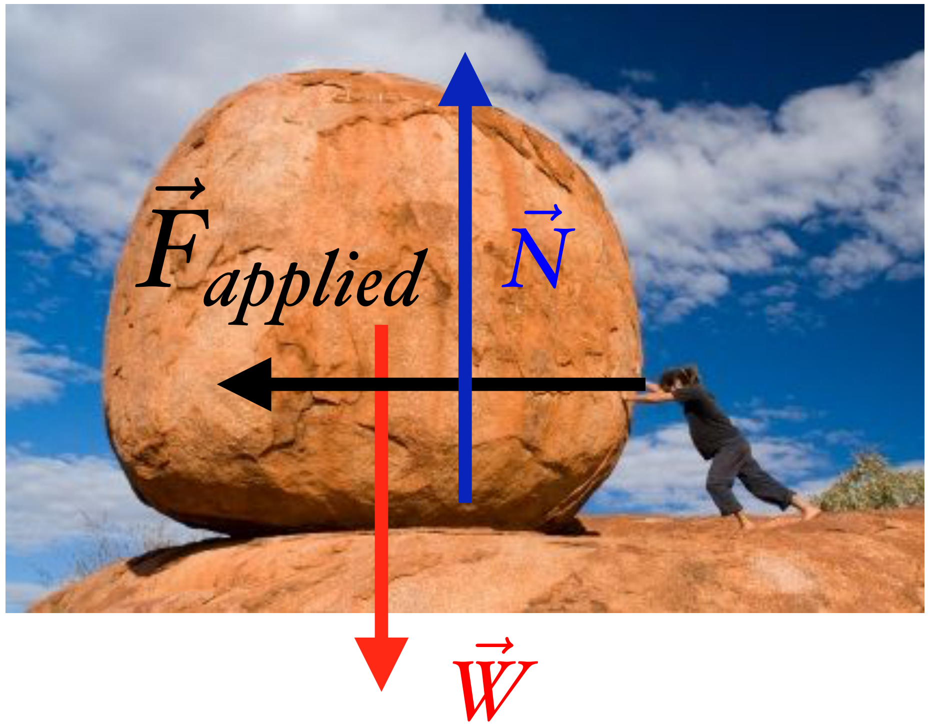


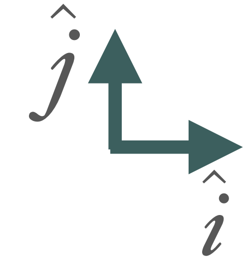
Let's try to figure it out!!

4th step: get the components

$$m\,a_x = F_{applied}$$

$$m\,a_y = N - m\,g$$

Solid friction



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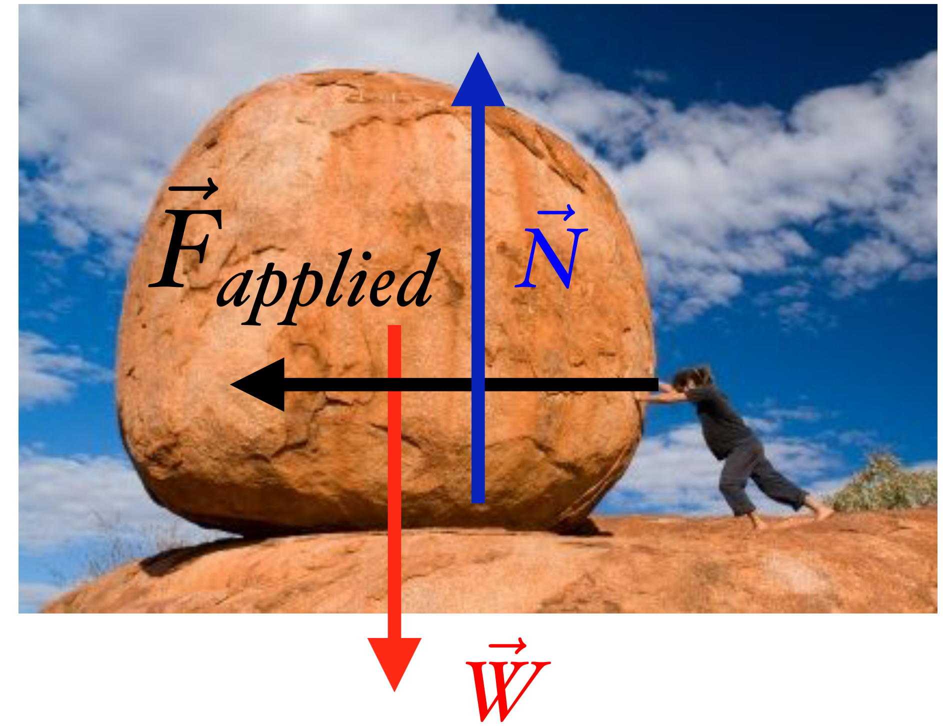


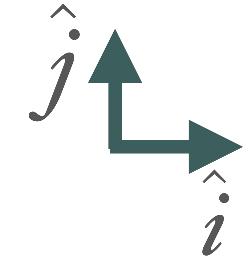
Let's try to figure it out!!

4th step: get the components

$$F_{applied} \neq 0 \implies a_x \neq 0$$

$$a_y = 0 \implies N = m\,g$$

Solid friction



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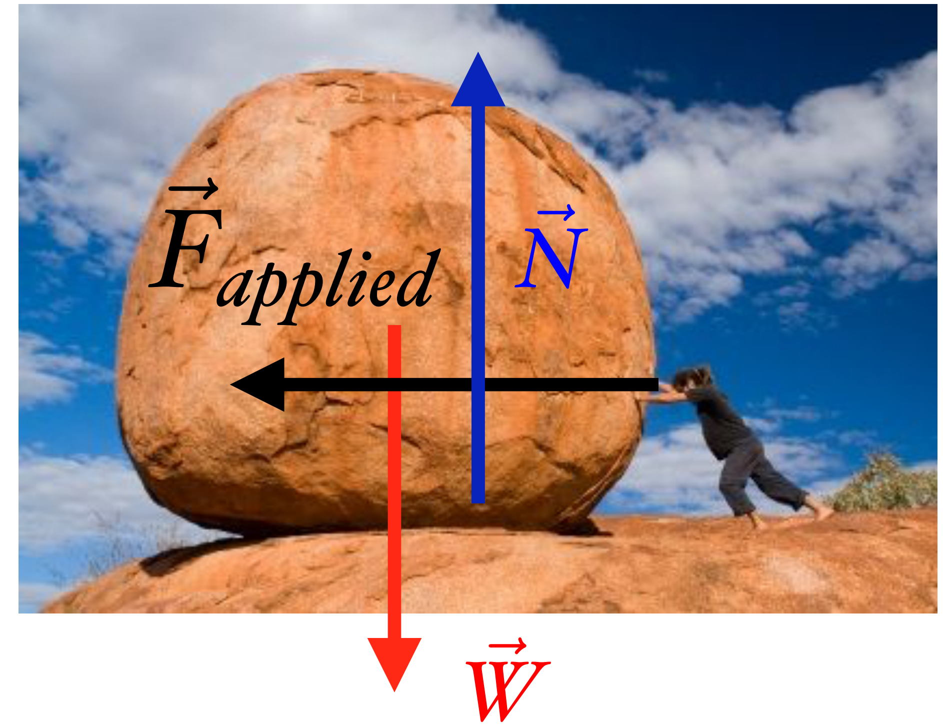

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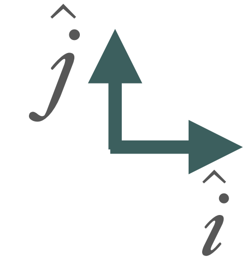

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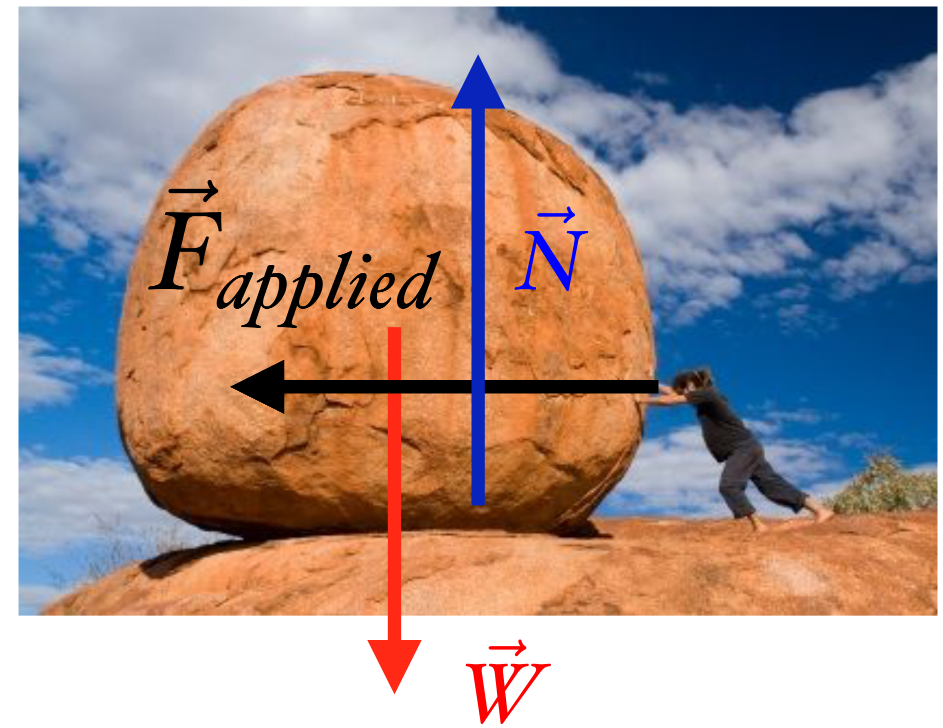


Let's try to figure it out!!

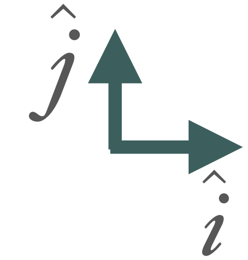
4th step: get the components

$$F_{applied} \neq 0 \implies a_x \neq 0$$

Really?
We must have missed something

$$a_y = 0 \implies N = m\, g$$

Solid friction



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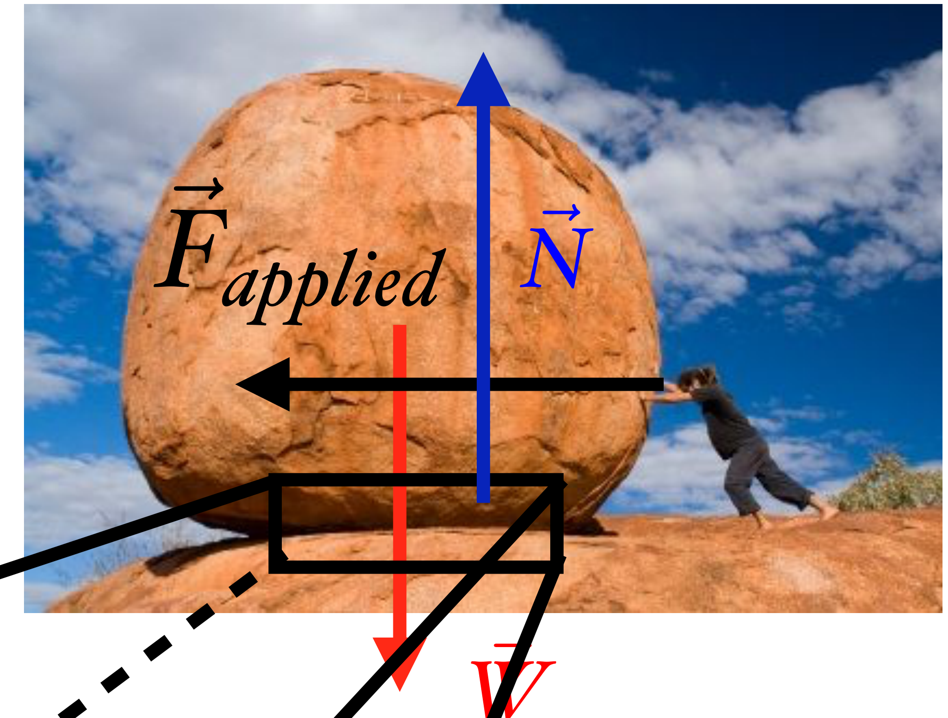
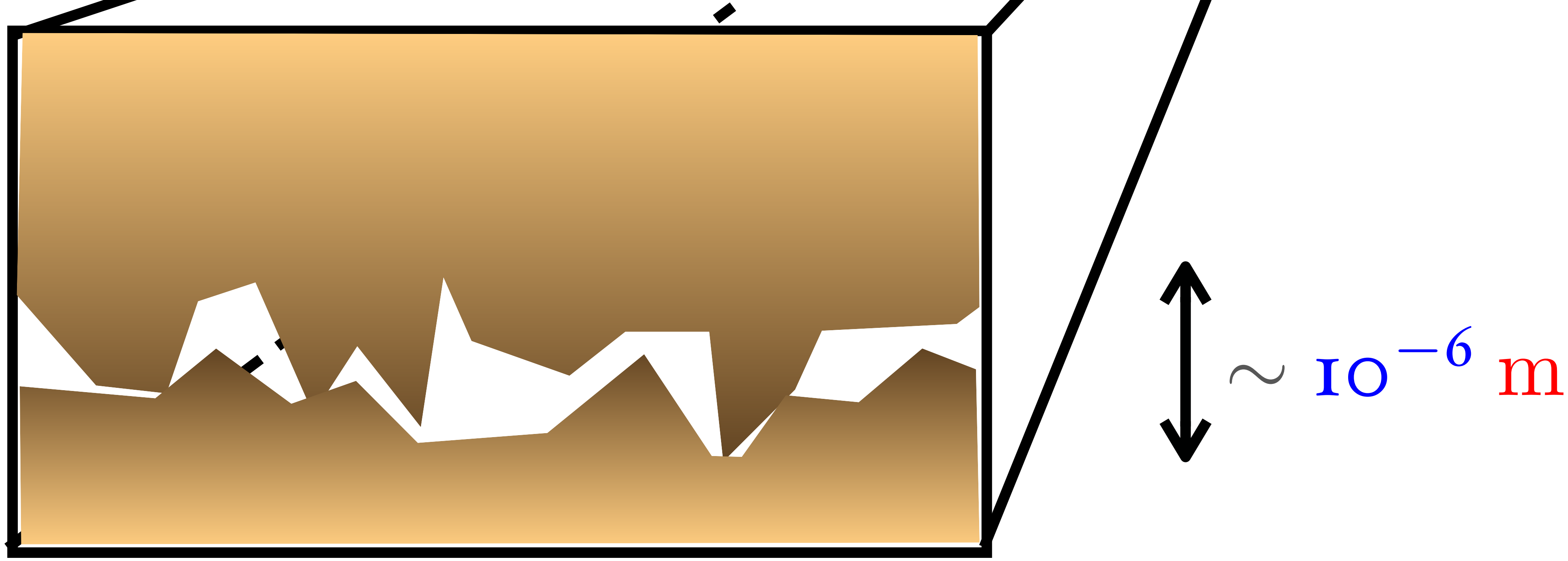
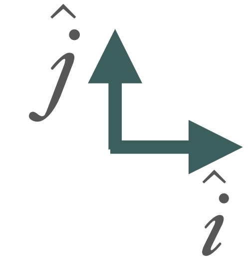


Let's try to figure it out!!

Rough surfaces



Solid friction



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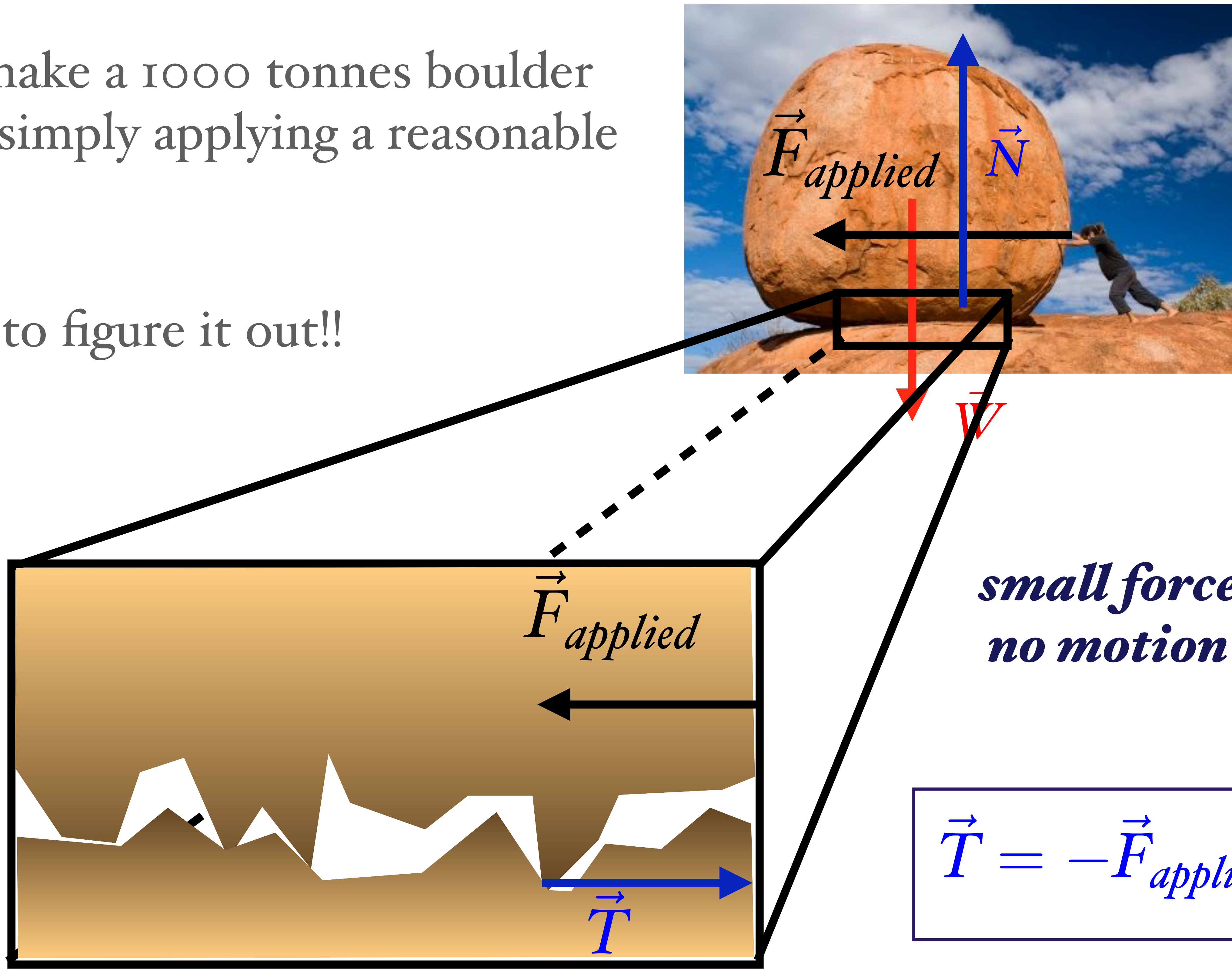
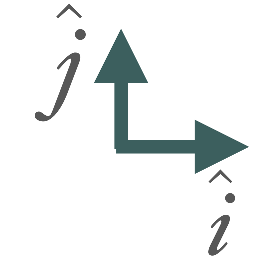


Rough surfaces

small force no motion

$$\vec{T} = -\vec{F}_{applied}$$

Solid friction



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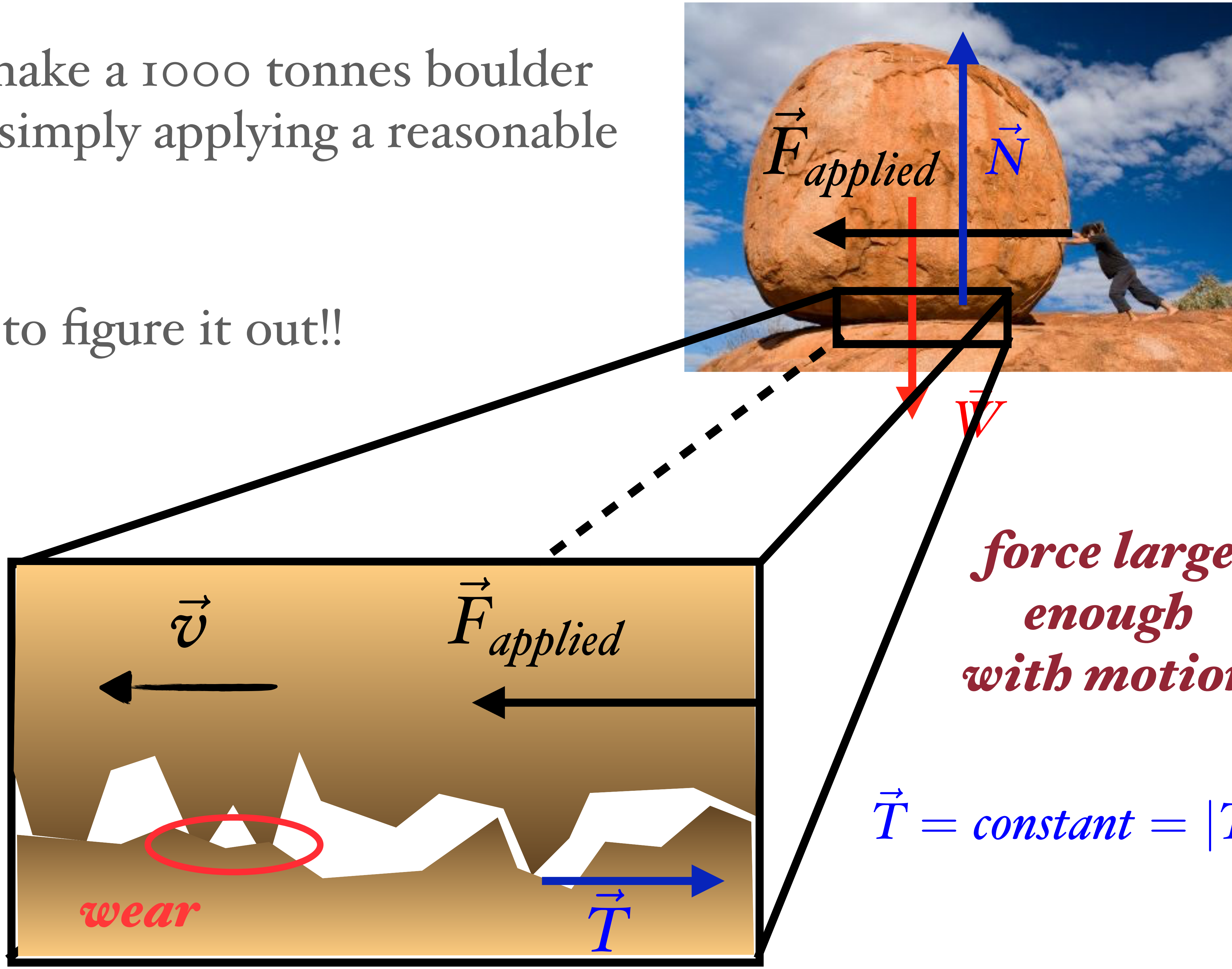


Rough surfaces

force large enough with motion

$\vec{T} = constant = |T|\,\hat{i}$

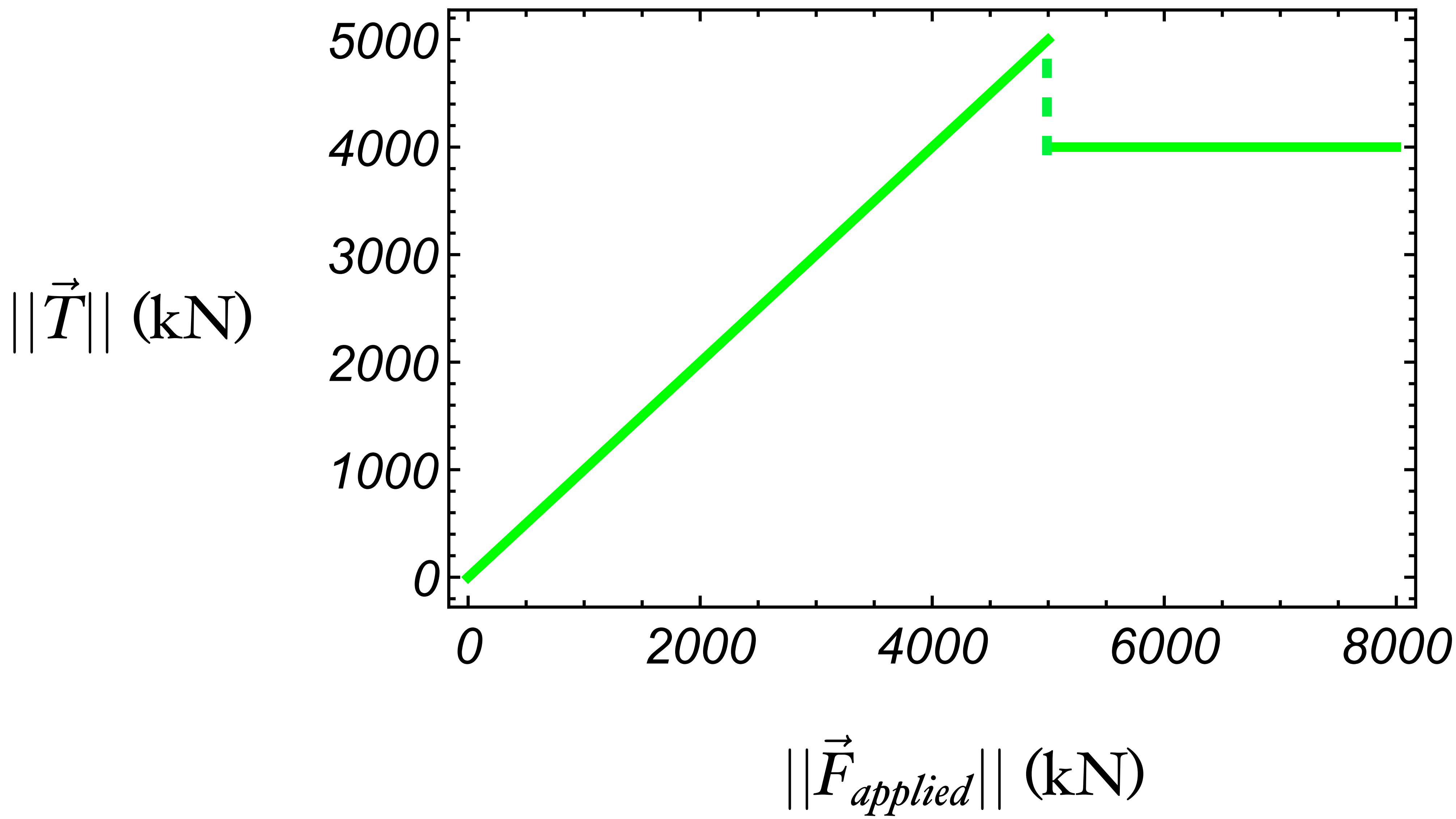
Solid friction

On rather soft grounds, the wear created by heavy objects is easily visible with the naked eye

Solid friction

Mathematical modelling

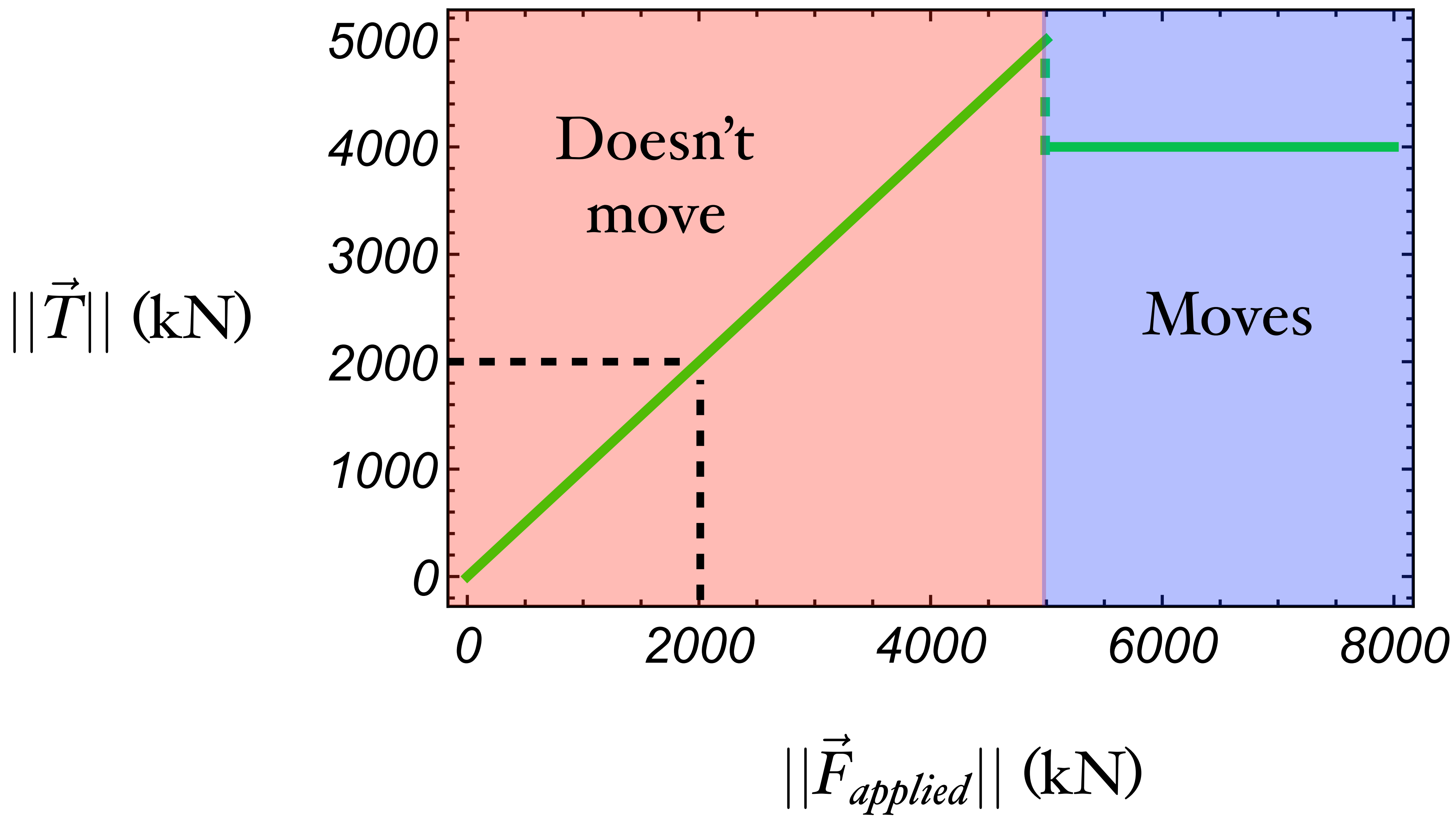
for a 1000 tonnes boulder



Solid friction

Mathematical modelling

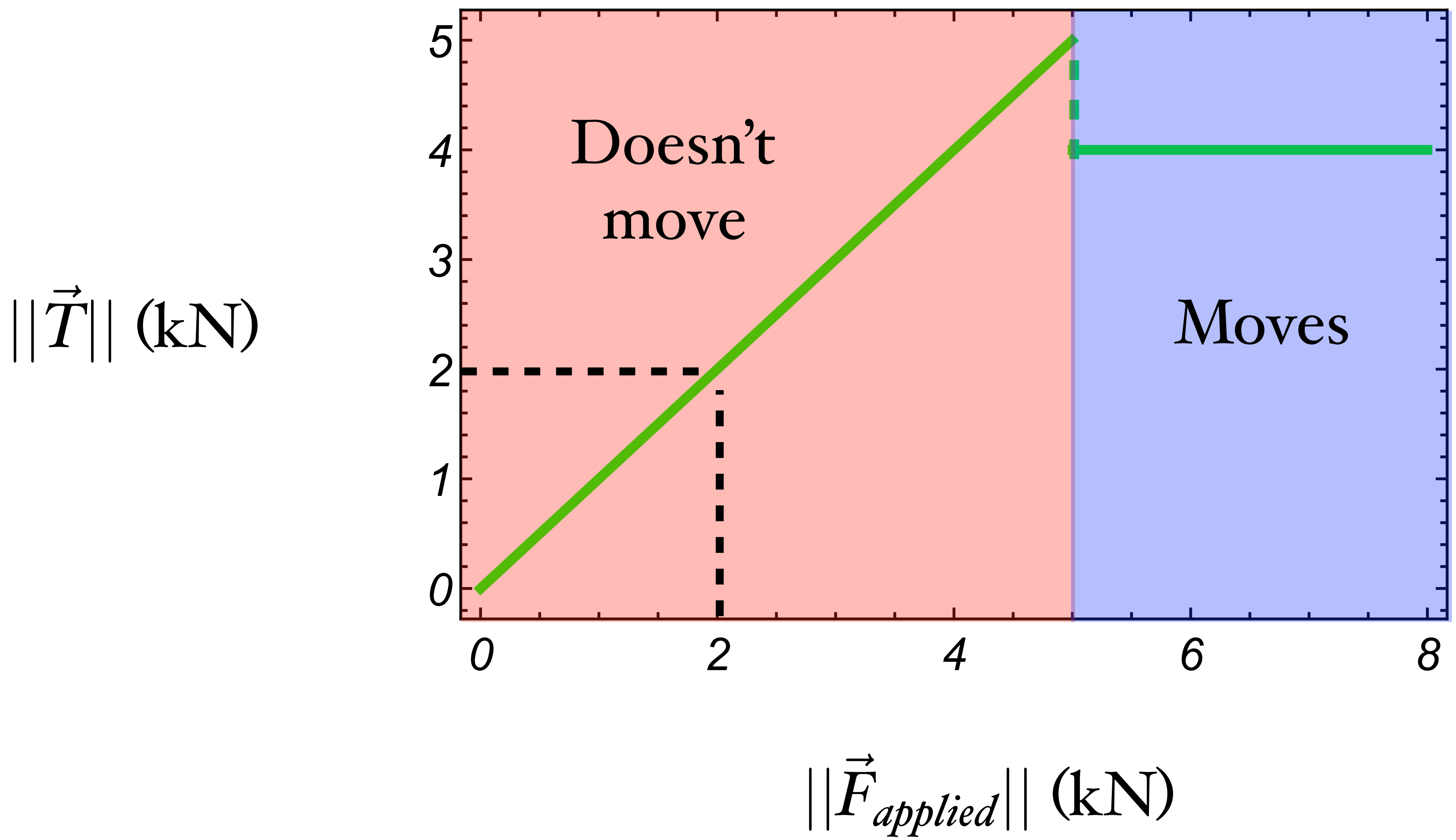
for a 1000 tonnes boulder



Solid friction

Mathematical modelling

for a 1 tonne boulder



Solid friction

Mathematical modelling

☆ If $\|\overrightarrow{F}_{applied}\| \leq \mu_s \|\overrightarrow{N}\|$ the tangential reaction $\overrightarrow{T}$ from the ground (*static friction force*) balances the applied force $\overrightarrow{F}_{applied}$,

$$\overrightarrow{T} = -\overrightarrow{F}_{applied}$$

and the object ***doesn't move***. The factor μ_s is called the ***static friction coefficient***.

Solid friction

Mathematical modelling

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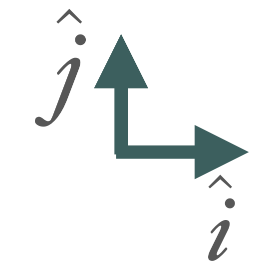
  and the object ***doesn't move***. The factor μ_s is called the ***static friction coefficient***.

- If $\|\overrightarrow{F}_{applied}\| > \mu_s \|\overrightarrow{N}\|$ the object moves. The tangential reaction $\overrightarrow{T}$ from the ground (*kinetic friction force*) opposes the motion with constant magnitude

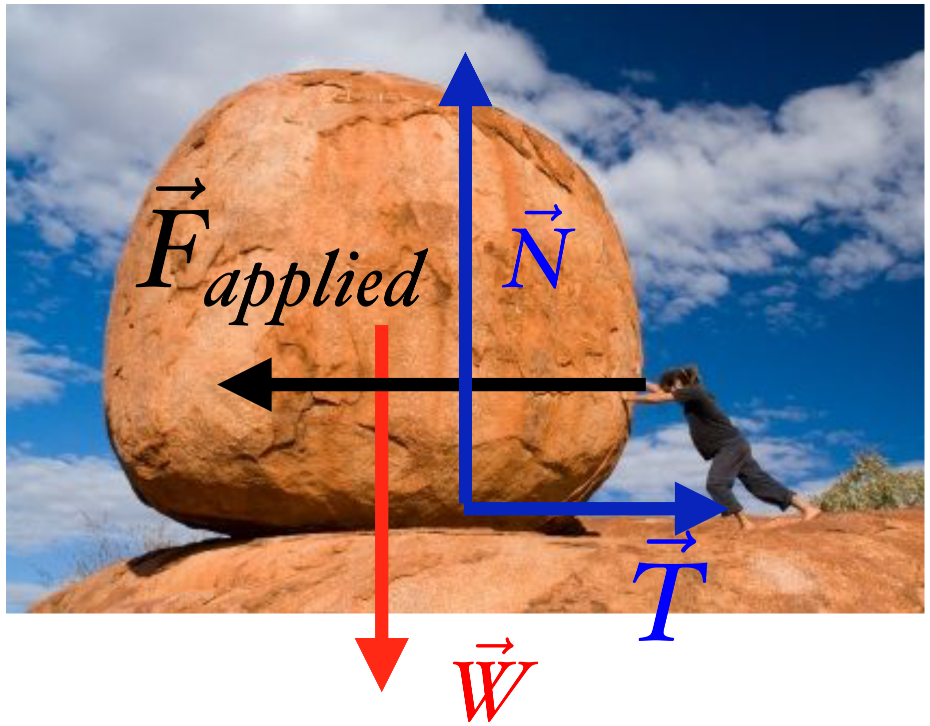
$$\|\overrightarrow{T}\| = \mu_k \|\overrightarrow{N}\|$$

  The factor $\mu_k \leq \mu_s$ is called the ***kinetic friction coefficient***.

Solid friction



Can we make a 1000 tonnes boulder move by simply applying a reasonable force given that the static friction coefficient is $\mu_s = 0.5$?



- If $||\overrightarrow{F}_{applied}|| \leq \mu_s ||\overrightarrow{N}||$ the boulder doesn't move
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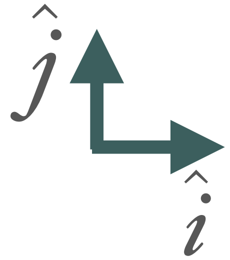
Solid friction



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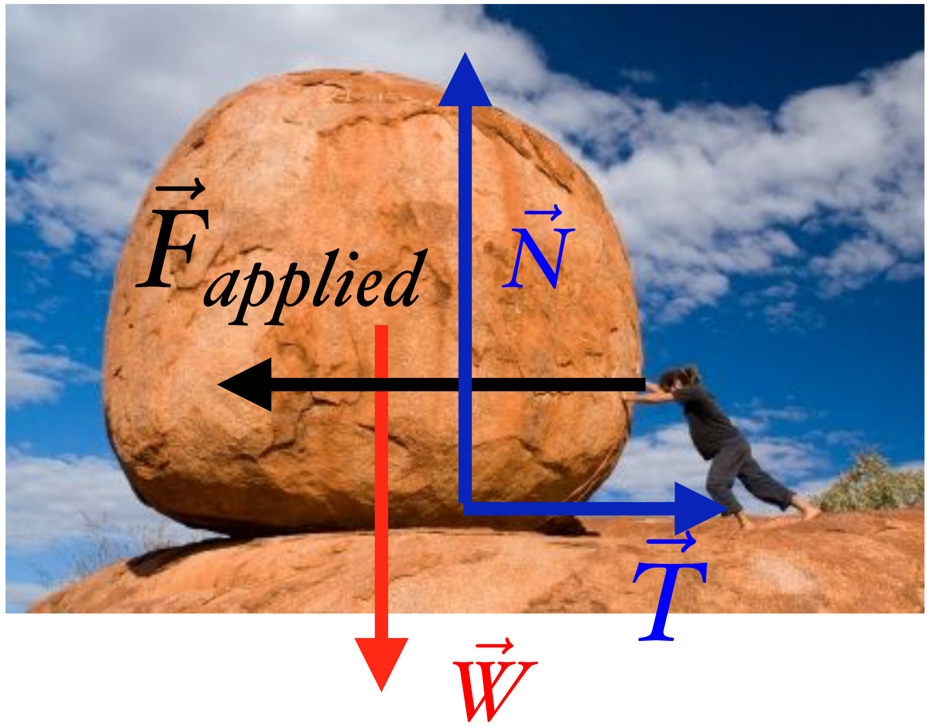


- If $\|\vec{F}_{applied}\| \leq \mu_s \|\vec{N}\|$ the boulder doesn't move
- If $\|\vec{F}_{applied}\| > \mu_s \|\vec{N}\|$ the boulder moves

$\vec{N} = -\vec{W} = -mg\hat{j}$, where $m = 1000^2 kg$ and $g = 9.8 m s^{-2}$

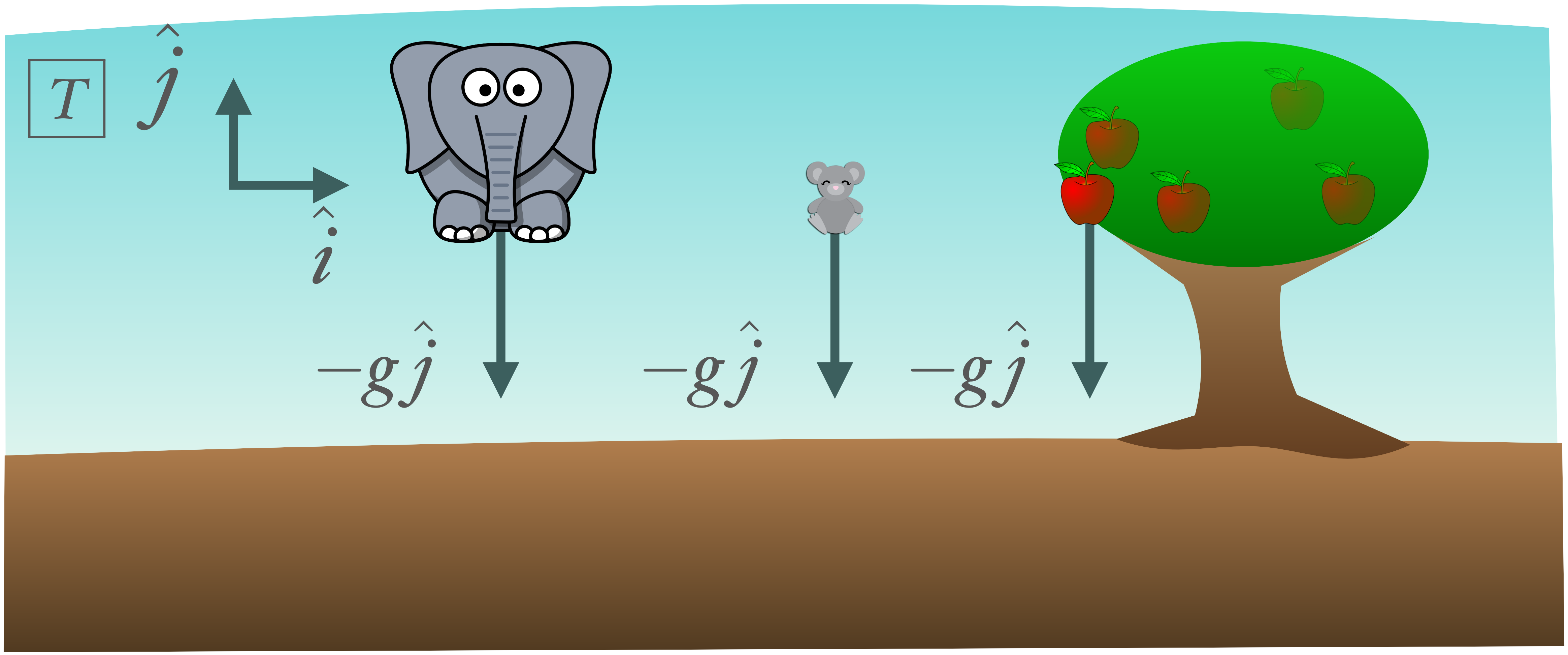
Hence, the boulder will move if

$$\|\vec{F}_{applied}\| > \mu_s mg = 0.5(1000^2 kg)(9.8 m/s^2) = 4.9(10^6) N$$

Enormous force!!!

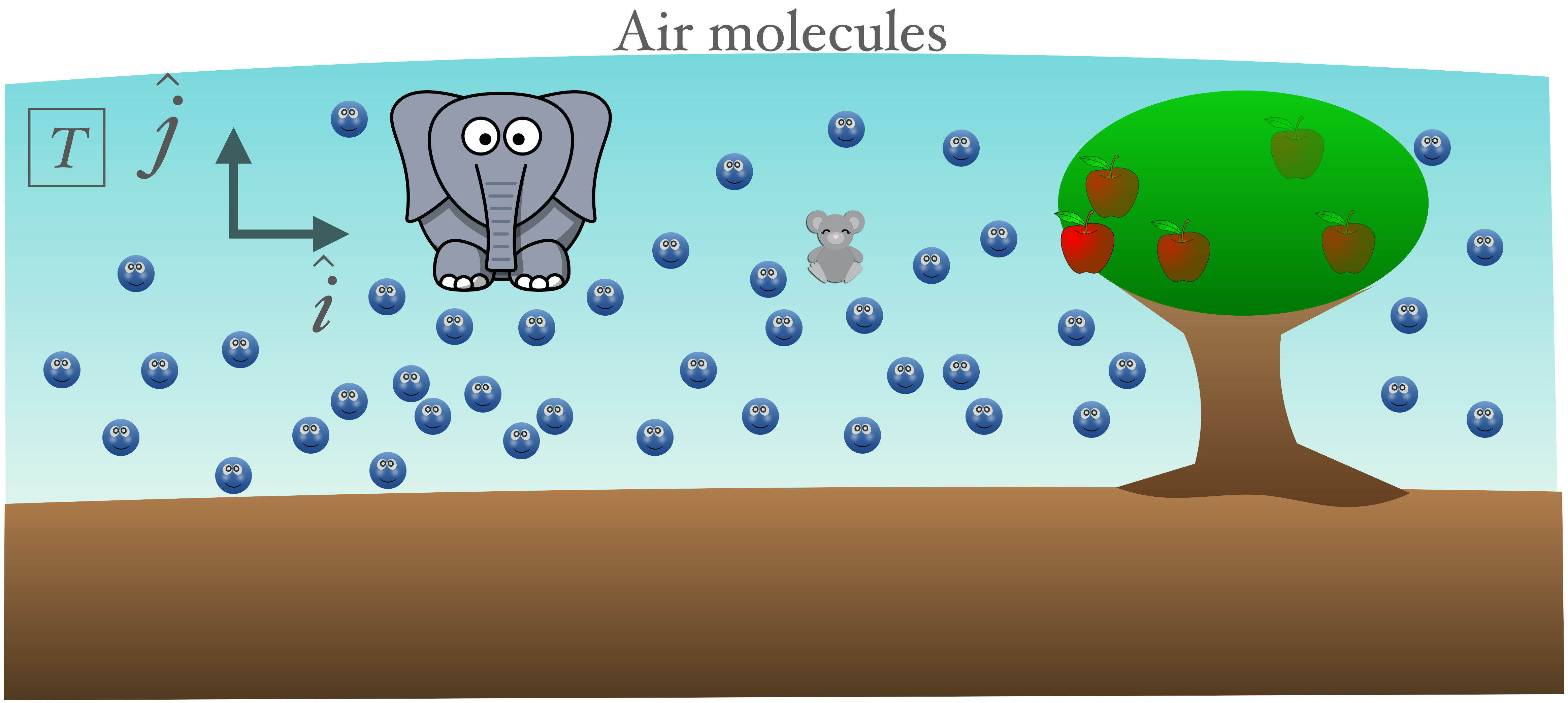
Fluid friction force

Going back to Galileo



Is it actually realistic to imagine that every object dropped from the same height will fall at the same speed on Earth in normal conditions?

Going back to Galileo



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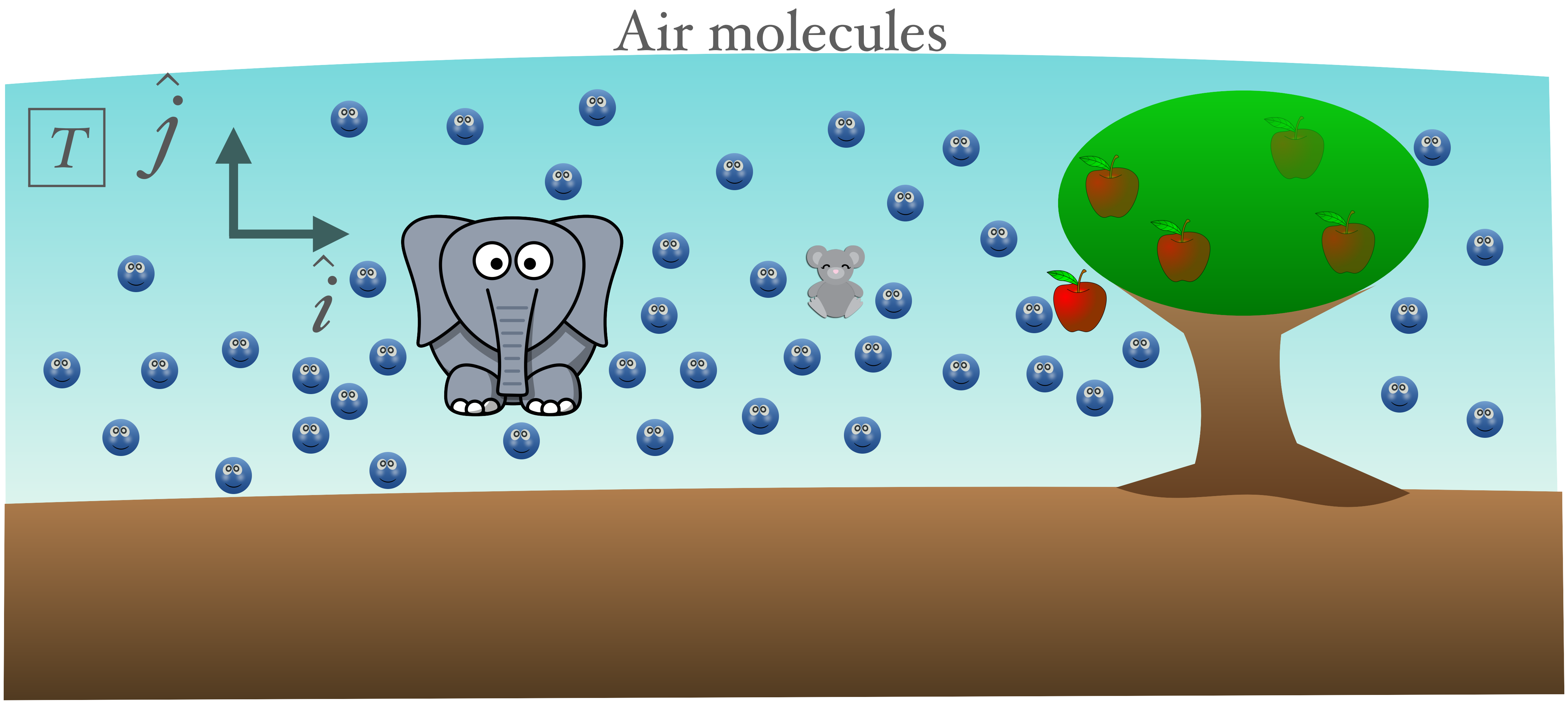

Is it actually realistic to imagine that every object dropped from the same height will fall at the same speed on Earth in normal conditions?

Upon moving down through air, all bodies are subject to a ***decelerating force*** whose final effect depends on their size

Fluid friction

Mathematical modelling

Every body moving through a fluid with velocity $\vec{v}$ relative to it is subject to a force $\vec{F}_{drag}$ opposing its motion and called ***drag force***

$$\vec{F}_{drag} = -\frac{1}{2}\,\rho\, C_D(||\vec{v}||)\, A\, ||\vec{v}||\vec{v}(t)$$

Fluid friction

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Fluid mass density

Drag coefficient

Cross sectional area of the body

Fluid friction

Mathematical modelling

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Very complicated

Fluid friction

Mathematical modelling

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$$\vec{F}_{drag} = -\frac{1}{2}\,\rho\, C_D(||\vec{v}||)\, A\, ||\vec{v}||\vec{v}(t)$$

In case of very small objects and/or very small velocities,

$$\frac{1}{2}||\vec{v}||\rho C_D(||\vec{v}||)A \approx \textit{constant} = \gamma \quad \text{(friction coefficient)}$$

i.e. the drag force is simply proportional to the velocity:

$$\boxed{\vec{F}_{drag} = -\gamma\vec{v}(t)}$$

We will only consider this equation in this module!

Energy

Motivations

* As we have seen them, Newton's laws of motion are ***enough*** to solve any dynamical problem, provided we are given an adequate model for the forces.

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  - *Could the 2nd law emerge from a deeper principle?*

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    * *The concept of force was not totally clear*
    * *Could the 2nd law emerge from a deeper principle?*

* Newton's laws are not able to capture all of our own sensations and apprehension of forces. For example:
    * *Why is it harder to climb stairs rather than walk on a flat surface if the same distance is traveled?*
    * *Is it possible to quantify the effort one needs to generate to perform a given mechanical task?*

The energy concept

"***It is harder to maintain a force of the same magnitude over a long distance than a short distance***"

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* In our society: "costly" means that whatever service or good we want, we will have to trade it against an amount of scarce resource that we call "money".

* In mechanics this "money" is called ***Energy***.

Kinetic energy

We consider a point object of mass m moving with velocity $\overrightarrow{v}(t)$

We define the ***kinetic energy*** of this point object as being:

$$K(t) = \frac{1}{2}m\|\overrightarrow{v}\|^2$$

If the object moves in one dimension with velocity $\overrightarrow{v}(t) = v_x(t)\hat{i}$ in a Galilean frame $(O, \hat{i})$, then

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$$K(t) = \frac{1}{2} m v_x(t)^2 = \frac{1}{2}(3kg)(6ms^{-1})^2 = 54\ kg \cdot m^2 \cdot s^{-2} = 54N \cdot m$$

How is energy spent or earned?

Let us consider a point object of mass m moving with velocity $\vec{v}(t) = v_x(t)\,\hat{i}$ in a Galilean frame $(O,\ \hat{i})$ and subject to a **constant** force $\vec{F} = F\,\hat{i}$

Since we are in a Galilean frame, Newton's 2nd law applies and, in 1D, can be directly written in component:

$$m\,a_x(t) = F \quad \Longrightarrow \quad m\,\dot{v}_x(t)\cdot v_x(t) = F\cdot v_x(t)$$

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If we consider that the position of the point object at time t_A is $x(t_A) = x_A$ and at time t_b is $x(t_B) = x_B$, then we derive

$$\int_{t_A}^{t_B} m\ v_x(t) \cdot \dot{v}_x(t) dt = \int_{t_A}^{t_B} F \cdot v_x(t) dt\ ,$$ which implies that

$$\frac{1}{2} m v_x(t_B)^2 - \frac{1}{2} m v_x(t_A)^2 = F(x_B - x_A) \quad \text{or} \quad K_B - K_A = F(x_B - x_A)$$

How is energy spent or earned?

The equation $K_B - K_A = F(x_B - x_A)$ is a specific case (for a constant force) of what is known as the ***work-energy theorem***.

It shows that when a constant force is applied on a point object over a non-zero distance, it changes its kinetic energy.

The quantity $F(x_B - x_A)$ is called the ***work done*** by the **constant** force F on the point object from points A to B.

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Remark 2: if F and $(x_B - x_A)$ have same signs, the force facilitates the motion of the object. It can then be interpreted as a ***monetary incentive*** to earn by the object to move from A to B

Work in one dimension

Work in one dimension

The general formula for the work done by a force $\overrightarrow{F}(x) = F(x)\hat{i}$ along a 1D path $\Gamma_{A\to B}$ from point A to point B is:

$$W(F\,|\,\Gamma_{A\to B}) = \int_{x_A}^{x_B} F(x)dx$$

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If we set $x = x(t)$, with t_A, t_B such that $x(t_A) = x_A$ and $x(t_B) = x_B$, then

$$dx = \frac{dx(t)}{dt}dt = v_x(t)dt \ , \quad \text{hence}$$

$$W(F\,|\,\Gamma_{A\to B}) = \int_{t_A}^{t_B} F(x(t)) \cdot v_x(t)dt$$

Work in one dimension

Example

Let us consider a path $\Gamma_{A \to B}$ that goes from x_A to x_C and then from x_C to x_B with $x_A < x_B < x_c$ and

$$F(x) = F, \quad for\ x \in [x_A, x_B],$$

$$F(x) = 2F, \ for\ x \in [x_B, x_C],$$

where F is a constant.

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where F is a constant.

Solution

$$\mathcal{W}(F|\Gamma_{A\to B}) = \int_{x_A}^{x_B} F\,dx + \int_{x_B}^{x_C} 2F\,dx + \int_{x_C}^{x_B} 2F\,dx$$

$$\boxed{\mathcal{W}(F|\Gamma_{A\to B}) = F \cdot (x_B - x_A)}$$

Work-energy theorem in 1D

Let us consider a point object of mass m moving with velocity $\vec{v}(t) = v_x(t)\,\hat{i}$ in a Galilean frame $(O,\,\hat{i})$ and subject to a force $\overrightarrow{F}(x) = F(x)\hat{i}$, whilst moving on a path $\Gamma_{A\to B}$ from point A to B

Then, according to Newton's 2nd law the following is true:

$$ma_x(t) = F(x(t)) \implies m\,\dot{v}_x(t)\,v_x(t) = F(x(t))\,v_x(t)$$

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If we consider that $x(t_A) = x_A$ and $x(t_B) = x_B$, then

$$\int_{t_A}^{t_B} m\,v_x(t)\cdot\dot{v}_x(t)dt = \int_{t_A}^{t_B} F(x(t))v_x(t)dt\ ,$$ which implies that

$$K_B - K_A = \mathcal{W}(F|\Gamma_{A\to B})$$

This equation is called the ***1D work-energy theorem***

Conservative forces

Conservative forces

A 1D force $\overrightarrow{F}(x) = F(x)\hat{i}$ is said to be ***conservative*** if either of the ***equivalent*** propositions is true:

- The work done by the force on a point object moving from a point A to a point B is ***independent of the path taken***

- The work done by the force on a point object along ***any closed*** path is ***zero***

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So, $W(F\,|\,\Gamma_{A\to B}) = \int_{x_A}^{x_B} F(x)dx = U(x_A) - U(x_B)$

Conservation of mechanical energy

If a 1D force F is conservative, then necessarily it can be associated to a function $U(x)$ such that $F(x) = -U'(x)$

In this case the work-energy theorem reads:

$$K_B - K_A = W(F\,|\,\Gamma_{A\to B}) = U(x_A) - U(x_B)$$

Reshuffling the terms yields:

$$\boxed{K_B + U(x_B) = K_A + U(x_A)}$$

There is ***conservation*** of what is called ***mechanical energy***

Mechanical and potential energy

If the forces acting on a system are conservative, then the ***mechanical energy*** $E = K + U$ is conserved.

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Physical dimension

$$[E] = [K] = [U] = [W] = M \cdot L^2 \cdot T^{-2}$$

The SI unit of energy/work is the ***joule*** (J)

$$1\text{ J} = 1\text{kg}\left(\frac{m}{s}\right)^2 = 1\text{N} \cdot m$$

“Deriving” Newton’s 2nd law in one dimension

Consider a point object of mass m in a Galilean frame subject to a conservative force $F(x)$ characterised by a potential energy $U(x)$

The mechanical energy is $$E = \frac{1}{2}\, m\, v_x(t)^2 + U(x)$$

Conservation of energy means that $\dot{E} = 0$

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product rule

$$2\frac{dv_x(t)}{dt} \cdot v_x(t)$$

chain rule

$$\frac{dU(x)}{dx} \cdot \frac{dx}{dt}$$

"Deriving" Newton's 2nd law in one dimension

Hence, $\dot{E} = 0$ implies $\left(m \dfrac{dv_x(t)}{dt} + \dfrac{dU(x)}{dx} \right) \cdot v_x(t) = 0$.

If we consider that $v_x(t) \neq 0$, we get that

$$m \frac{dv_x(t)}{dt} = - \frac{dU(x)}{dx}$$

and since $-\dfrac{dU(x)}{dx} = F(x)$, we derive $\boxed{ma_x(t) = F(x)}$

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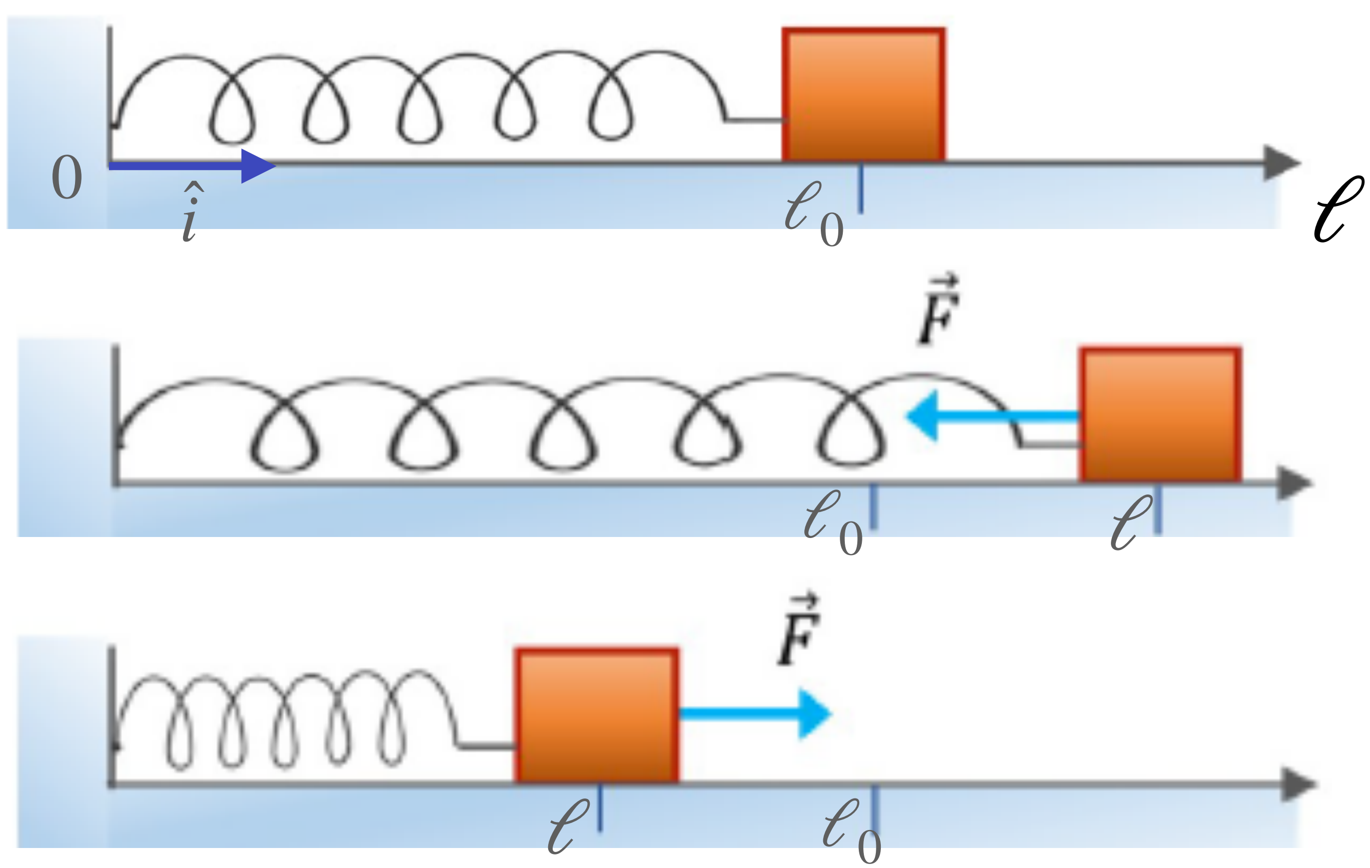
Notice that this derivation of Newton’s 2nd law works only for a point object subject to a conservative force.

Forces exerted by a spring

Hooke's law

We consider an object of mass m on a frictionless horizontal surface attached to the end of a spring whose rest length is ℓ_0. When the block is displaced from its equilibrium position, the spring exerts a restoring force $\overrightarrow{F} = F(\ell)\hat{i}$.



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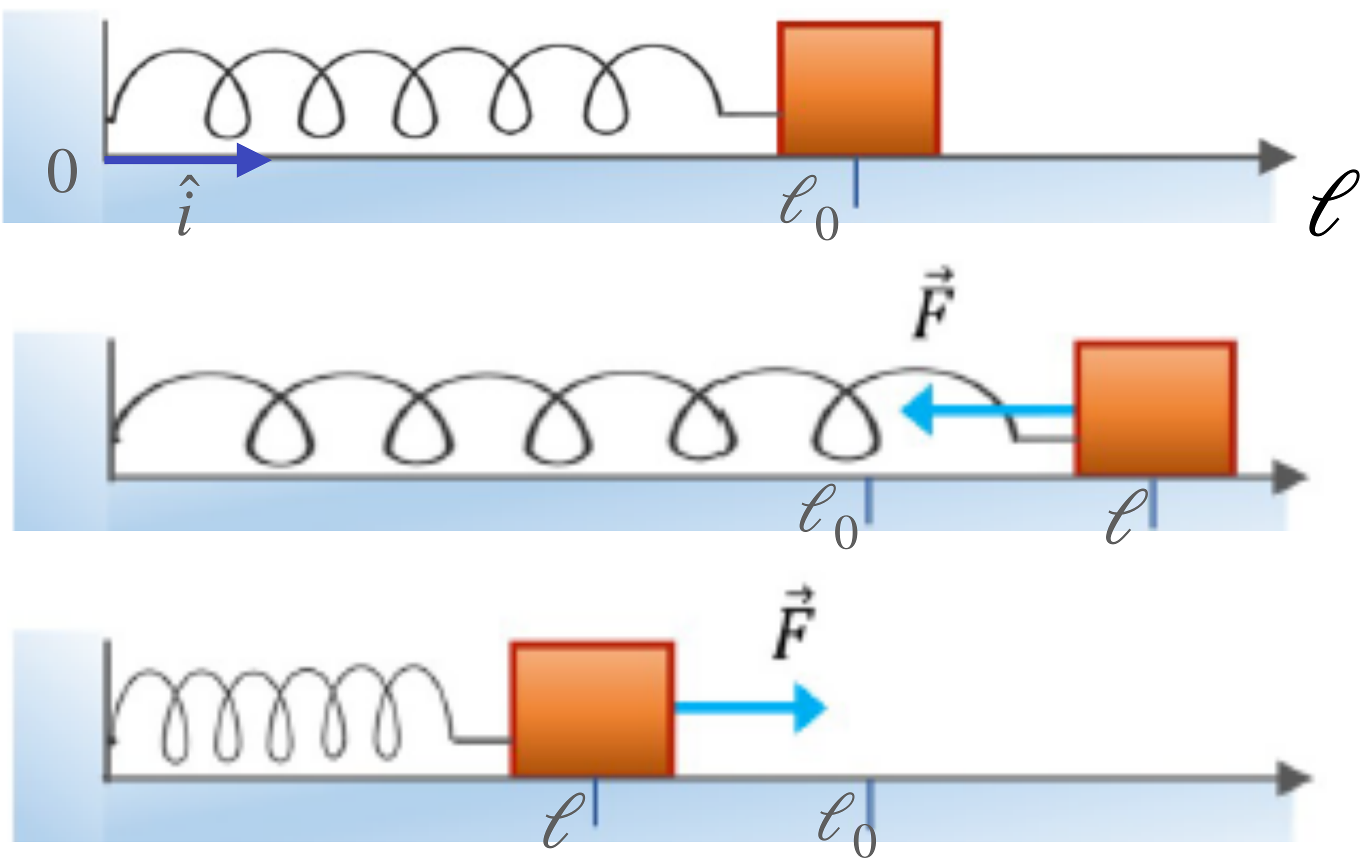

Hooke's Law: $F(\ell) = -k(\ell - \ell_0)$, for a constant $k > 0$.

The constant k (with units N/m) is a measure of the stiffness of the spring.

Hooke's law

<u>Exercise</u>

Show that the spring force $\overrightarrow{F} = F(\ell)\hat{i}$, with $F(\ell) = -k(\ell - \ell_0)$ is a conservative force with potential energy

$$U(\ell) = \frac{1}{2}k(\ell - \ell_0)^2$$

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$$U(\ell) = \frac{1}{2}k(\ell - \ell_0)^2$$

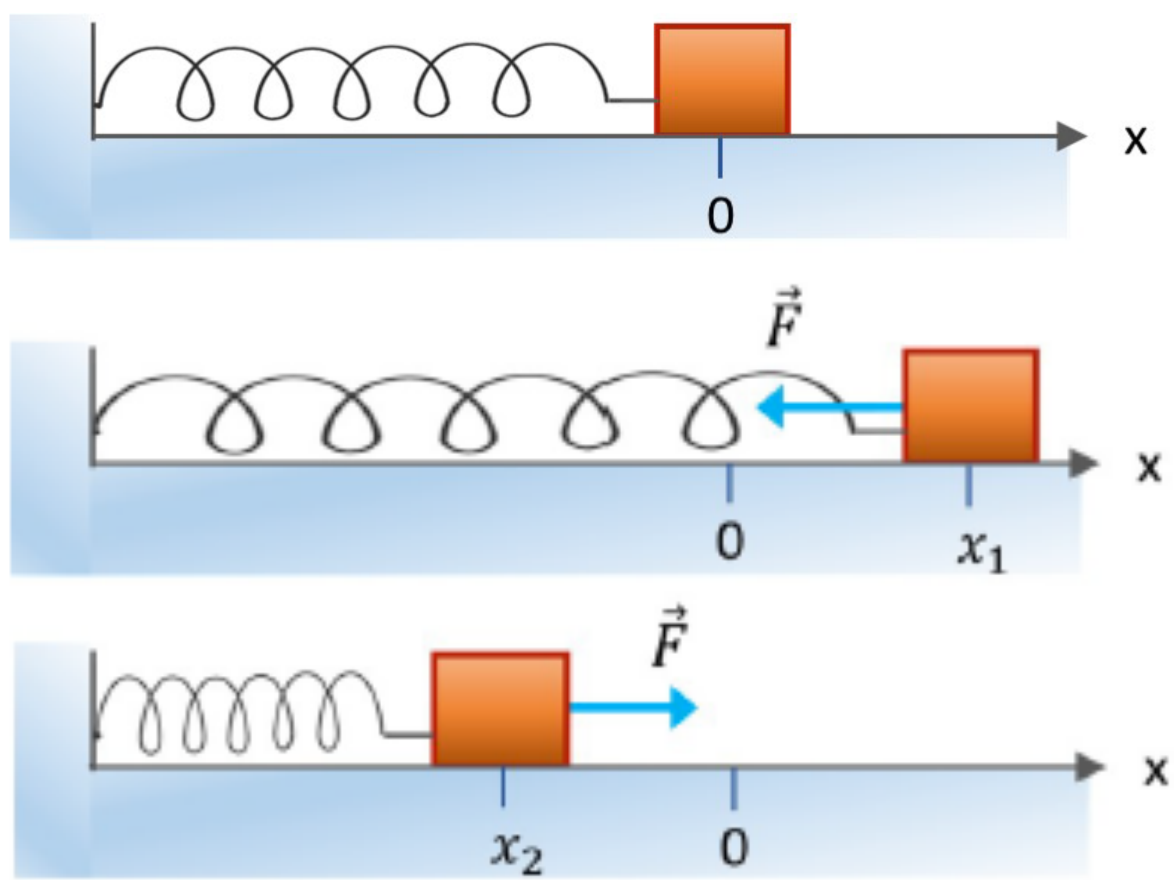
Solution

$$\frac{dU(\ell)}{d\ell} = k(\ell - \ell_0), \text{ so } F(\ell) = -\frac{dU(\ell)}{d\ell}.$$

Hooke's law

Remark

If we set the origin of the reference frame at ℓ_0 and x denotes the displaced from the equilibrium position, i.e. $x = \ell - \ell_0$,



then the spring force can be expressed as $\overrightarrow{F} = F(x)\hat{i}$, with $F(x) = -kx$.