The Laws of Motion

The three laws of Newton

- **Law 1**: *A body continues in its state of rest, or in uniform motion in a straight line (motion with constant velocity), unless acted upon by a force.*

- **Law 2**: *The acceleration produced when a force acts is directly proportional to the force and takes place in the direction in which the force acts.*

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- **Law 3**: *To every action there is an equal and opposite reaction: whenever one object exerts a force on another object, the second object exerts an equal in magnitude and opposite in direction force on the first.*

Newton's First Law

the law of inertia

Deciphering Newton's 1st law

Newton rightly pointed out that if **both** the full relativity of motion **and** his first law are true at the same time, then contradictions arise.

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Take for example two free falling objects: an apple (A) and a stone (S) which are dropped from the top of the Tower of Pisa with zero initial velocity.

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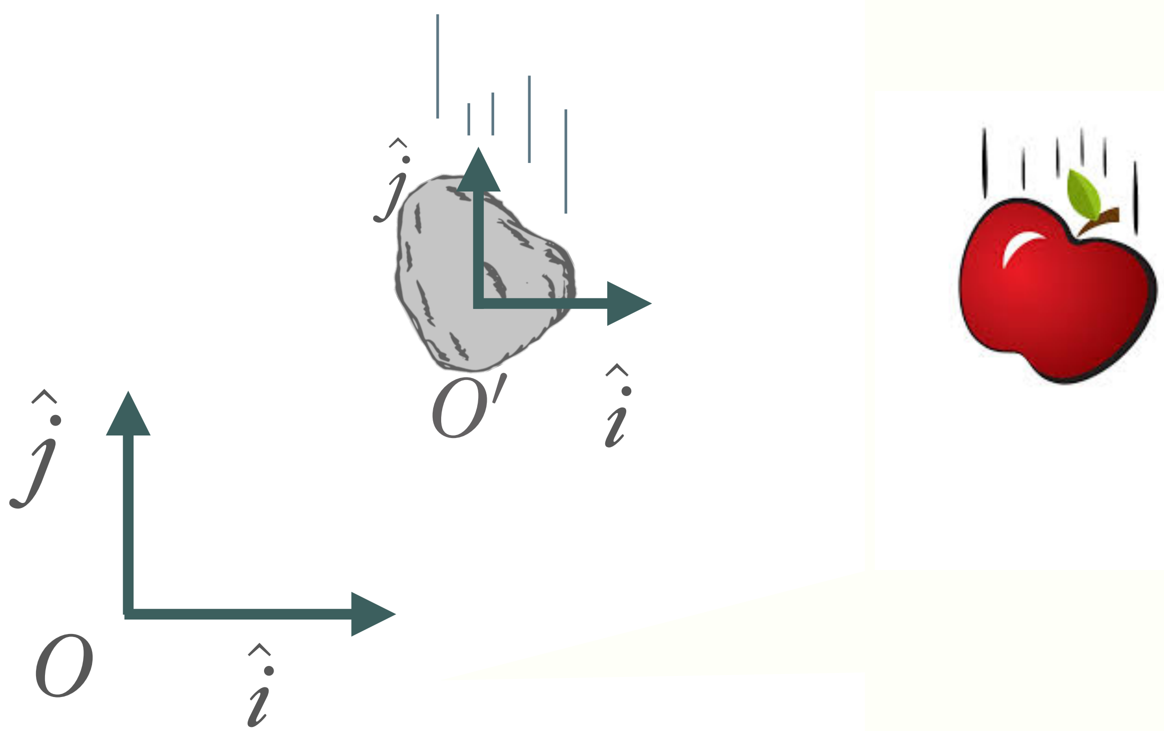

We can consider a frame of references $F(O, \hat{i}, \hat{j})$ on the ground and a second one $F(O', \hat{i}, \hat{j})$ attached to the stone.

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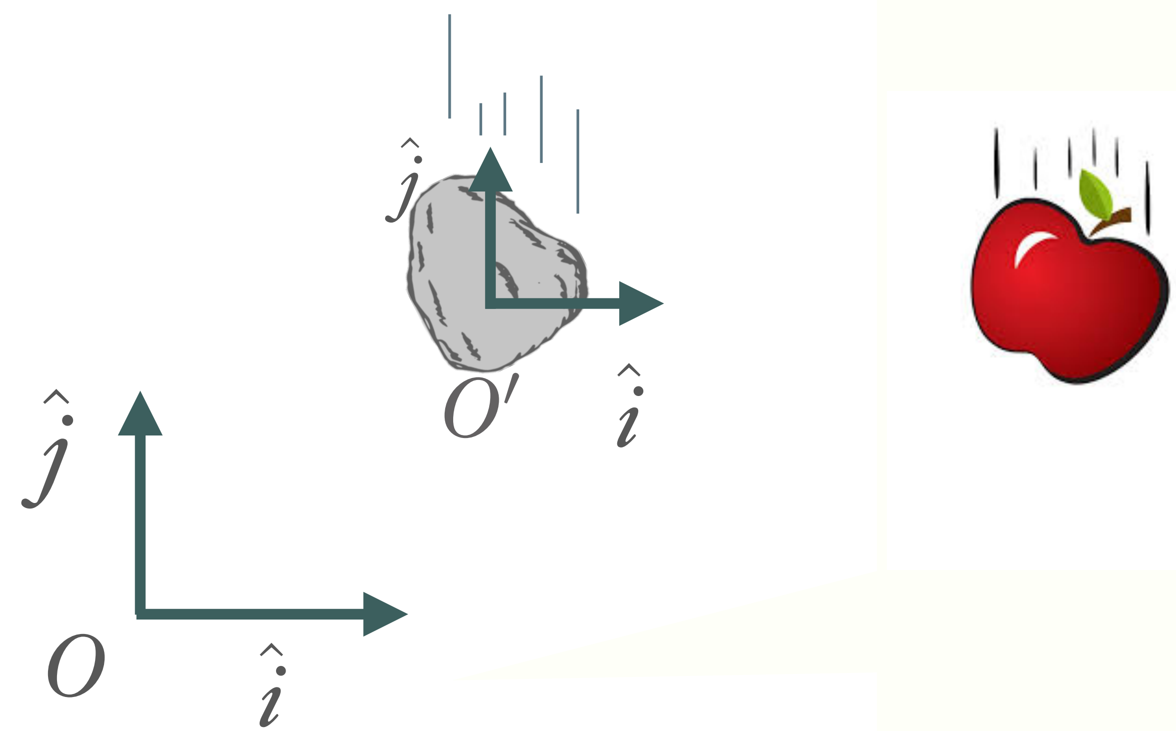

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Galileo tells us that $\overrightarrow{v}(A\,|\,O) = \overrightarrow{v}(S\,|\,O)$. Also by the law of composition of velocities $\overrightarrow{v}(A\,|\,O) = \overrightarrow{v}(S\,|\,O) + \overrightarrow{v}(A\,|\,O')$, hence

$\overrightarrow{v}(A\,|\,O') = 0$, i.e. the apple remains at rest with respect to the frame of reference of the stone.

Deciphering Newton’s 1st law

Newton rightly pointed out that if **both** the full relativity of motion **and** his first law are true at the same time, then contradictions arise.

Any motion, even under impressed forces, can look like a uniform motion in a special chosen frame of reference.

This fact seems to contradict the fist law of motion!

Deciphering Newton's 1st law

Newton's first law holds only with respect to a certain very special class of frames of reference which are called ***inertial frames of reference*** or ***Galilean frames of reference***.

An inertial frame of reference is, by definition, one in which isolated objects, not subject to forces, move at constant velocity, i.e. a frame of reference in which Newton's first law holds.

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By the law of composition of velocities any frame in uniform translation (moving with constant velocity and not rotating) with respect to an inertial frame of reference will also witness a uniform motion in absence of external forces, hence it constitutes an inertial frame of reference as well.

Deciphering Newton’s 1st law

Reformulation of the first law:

There exists a class of reference frames, called inertial or Galilean frames, such that for any Galilean frame F and any moving point object M the following holds:

If $\vec{a}(M|F) = 0$, then no net motive force is acting on M and conversely, if no net motive force is acting on M then $\vec{a}(M|F) = 0$.

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The special principle of relativity

The laws of mechanics are the same with respect to all Galilean frames of reference.

Deciphering Newton's 1st law

Example of application of the fist law:

We consider a Galilean frame $(O,\ \hat{i},\ \hat{j})$ in which we observe the motion of a point object with the following position vector

$$\vec{r} = (3\ \mathrm{cm})\,\hat{i} + (-87\ \mathrm{cm}\cdot\mathrm{s}^{-1})\,t\,\hat{j}$$

Question: Is there any net force acting on this point object?

Deciphering Newton's 1st law

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$$\vec{v} = (-87\ \mathrm{cm}\cdot\mathrm{s}^{-1})\ \hat{j}$$

The velocity vector is ***constant***, therefore, by the 1st law of Newton, there is ***no net force*** acting on the point object.

Newton's Third Law

the law of action-reaction

Deciphering Newton's 3rd law

To every action there is an equal and opposite reaction: whenever one object exerts a force on another object, the second object exerts an equal in magnitude and opposite in direction force on the first.

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Forces are ***vector quantities*** with a magnitude and a direction. They are often denoted $\overrightarrow{F}$ (not be confused with reference frames!!!)

Let us consider 2 bodies/objects 1 and 2. Being in mutual interaction means that they exert forces on each other. We denote $\overrightarrow{F}_{1\to 2}$ the force of object 1 on object 2 and $\overrightarrow{F}_{2\to 1}$ the force of object 2 on object 1.

Deciphering Newton's 3rd law

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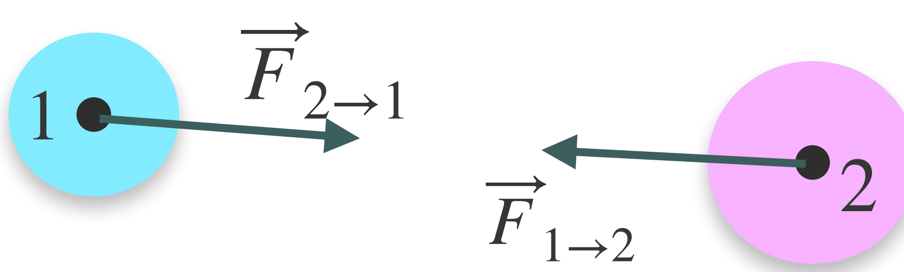

Deciphering Newton's 3rd law

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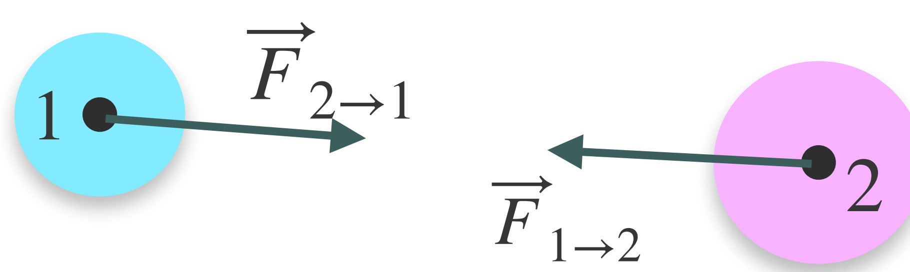

Newton's 3rd law: $\overrightarrow{F}_{1\to 2} = -\overrightarrow{F}_{2\to 1}$

Newton's Second Law

the fundamental law of dynamics

Deciphering Newton's 2nd law

- ***The acceleration produced when a force acts is directly proportional to the force and takes place in the direction in which the force acts.***

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Reformulation of the 2nd law

1. Let $\overrightarrow{a}(t)$ be the acceleration of a point object M as observed in a Galilean frame $(O, \hat{i}\,.\,\hat{j})$
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Inertial mass

$$\boxed{m\overrightarrow{a}(t) = \overrightarrow{F}}$$

Deciphering Newton's 2nd law

Force and mass

The concept of the force action that we have introduced is new and was not present in kinematics.

Yet, in Newtonian mechanics forces exist, we can talk about them, about their action on objects…but without being able to clearly define them; a bit like space and time in fact.

Alternatively, the 2nd law enables the force concept to be derived from space, time and yet another concept the ***inertial mass*** (or just mass).

Deciphering Newton's 2nd law

- The (inertial) **mass** of an object is a (scalar) quantity associated with the amount of material that is present in the object. It is that property of an object that specifies how much resistance an object exhibits to changes in its velocity

- Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it.

- In mechanics, we consider the mass as a primitive concept akin to space and time.

Physical dimensions of mass and force

The inertial mass is a primitive concept like space and time and must be measured with respect to a mass standard (unit) u_m .

Any mass can then be expressed as $m = ru_m$, where r is a positive number and u_m a unit of mass.

$$\boxed{\text{The SI unit of mass is the kilogram (kg)}}$$

- **Dimension:** $[m] = [u_m] = M$

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The three fundamental quantities in mechanics are length, mass and time. All other quantities can be expressed in terms of these three.

Physical dimensions of mass and force

The force concept then derives from the inertial mass, space and time concepts by application of the second law: $\vec{F} = m\vec{a}$

- **Dimension**: $[\vec{F}] = [m\vec{a}] = [m][\vec{a}] = M \cdot L \cdot T^{-2}$

The SI unit of force is the Newton (N)

$$1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$$

Newton's 2nd law

Example

A 3kg object is moving with velocity

$$\overrightarrow{v}(t) = (2t\ ms^{-2})\hat{i} + (5t\ ms^{-2})\hat{j}.$$

Find the resultant force (*net force*) acting on the object.

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$$\overrightarrow{a}(t) = \dot{\overrightarrow{v}}(t) = (2\hat{i} + 5\hat{j})m/s^2\,.$$

$$\overrightarrow{F} = m\overrightarrow{a} = (3kg)(2\hat{i} + 5\hat{j})m/s^2 = (6\hat{i} + 15\hat{j})N$$

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The **net force** is the vector sum of forces acting on a particle or object. The net force is a single force that replaces the effect of the original forces on the particle's motion.

Introducing the notion of weight

We call ***weight*** and denote $\overrightarrow{W}$, the ***force impressed*** on a body pulling it towards the ground on Earth.

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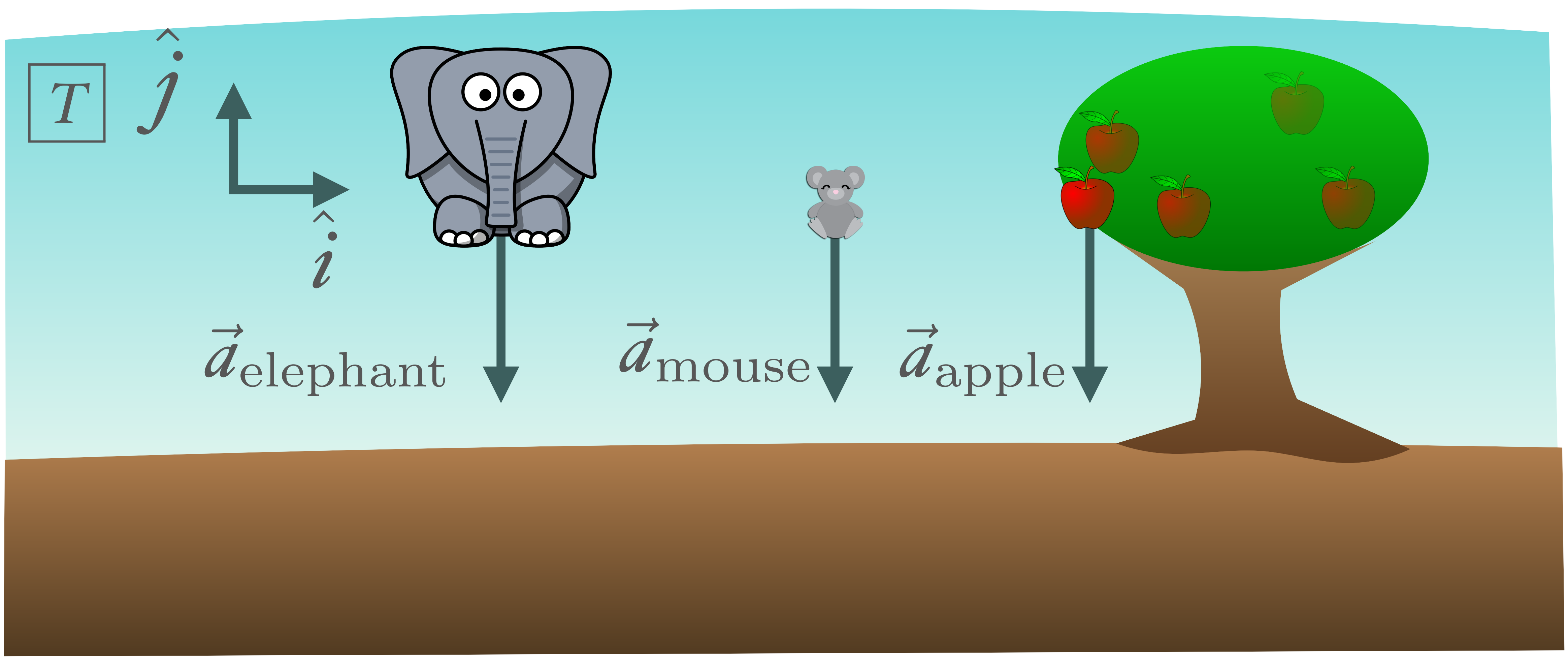
- According to Galileo: "*Provided air resistance can be neglected, all bodies let go from the same place* ***fall on Earth with the same acceleration*** *as observed from a terrestrial frame T*"



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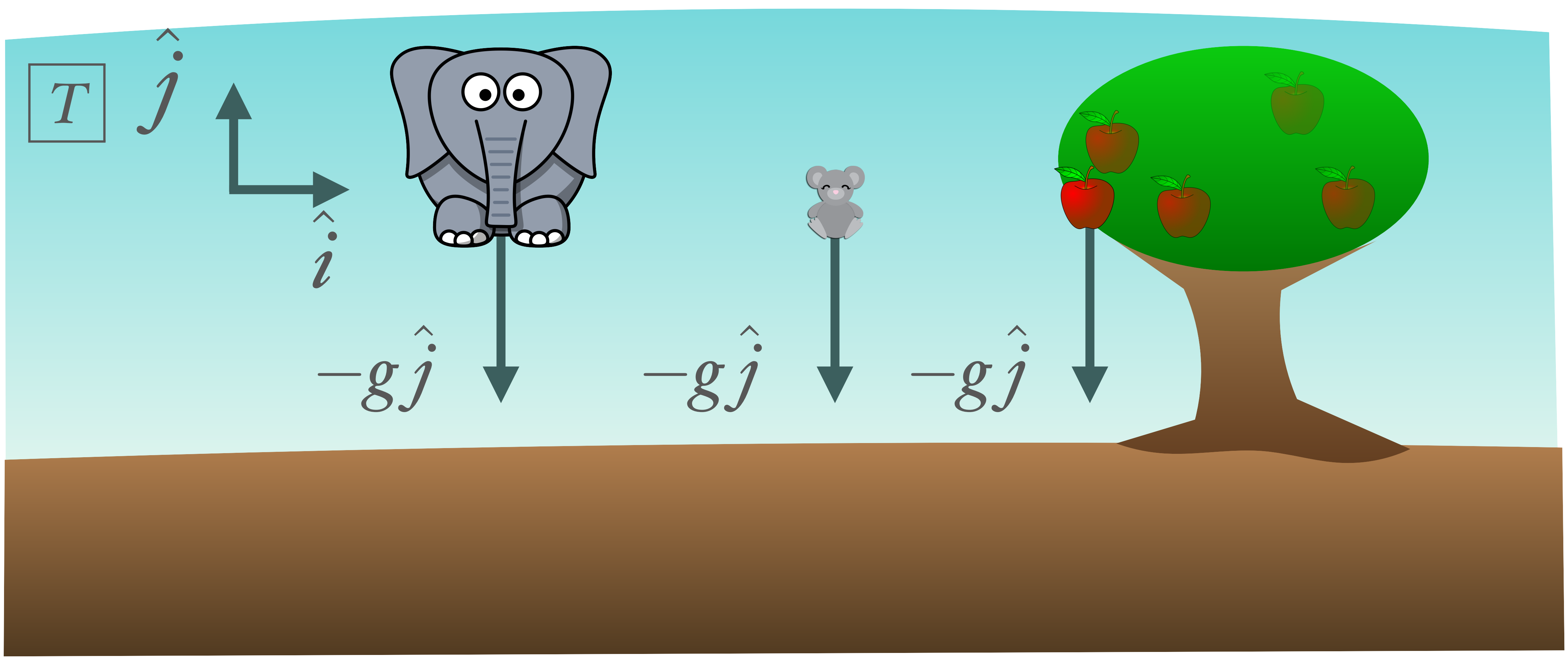

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- According to Newton's 2nd law, if T is a Galilean frame then

$$\overrightarrow{W} = m\overrightarrow{a}$$

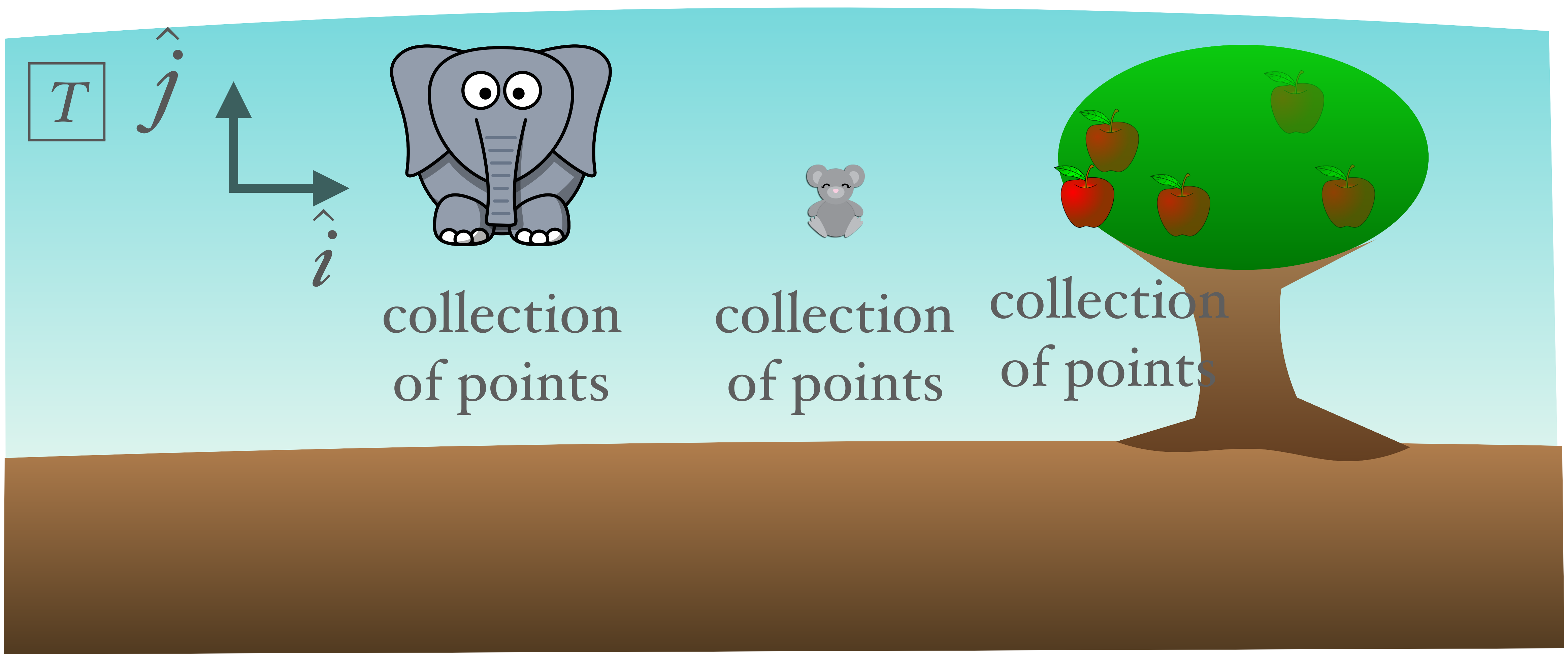
- By comparison with Galileo's proposal we get

$$\boxed{\overrightarrow{W} = -mg\hat{j}}$$

Why the concept of point object works

System comprising multiple parts

The laws of Newton concern the motion of point objects on which forces are acting. In reality, however a real object is not a point but an extended object; in other words a ***collection of points***



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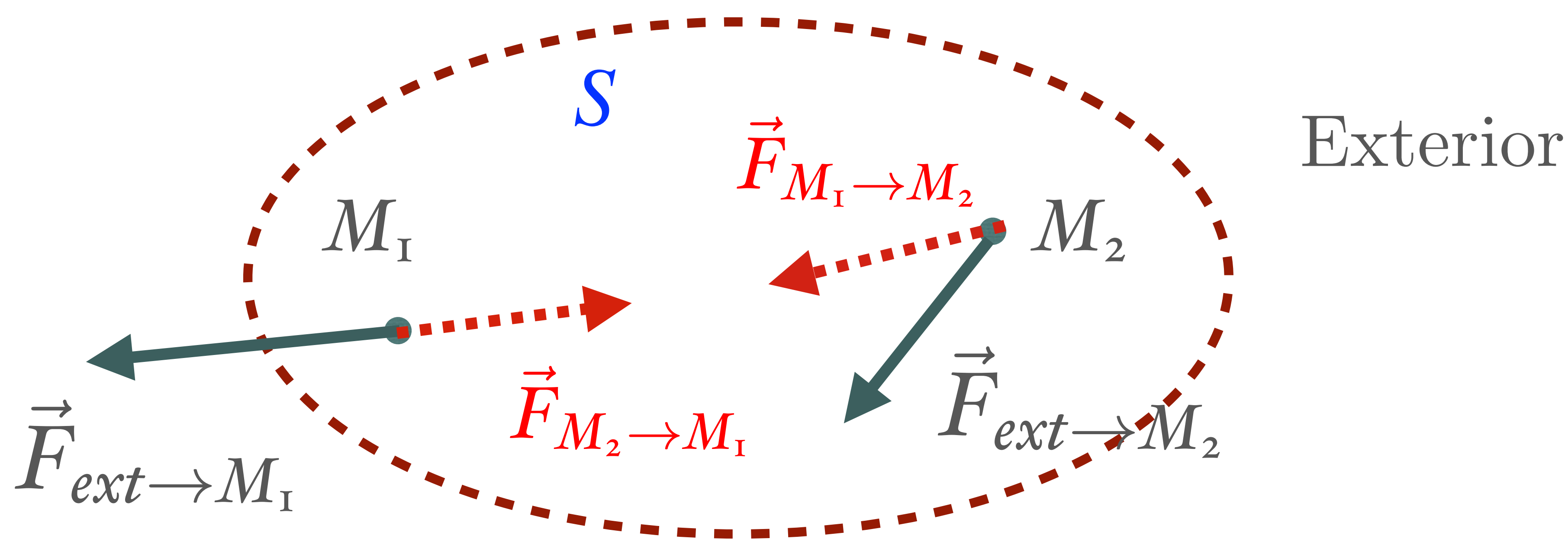
We consider a system S comprising two point objects M_1 and M_2 with respective masses m_1 and m_2 each subject to external forces $\overrightarrow{F}_{ext\to M_1}$ and $\overrightarrow{F}_{ext\to M_2}$ and moreover having a mutual interaction.



System comprising multiple parts

If the dynamics of the system S is observed from a Galilean frame of reference, then Newton's 2nd law applies for each part of the system:

$$m_1\overrightarrow{a_1} = \overrightarrow{F}_{ext\to M_1} + \overrightarrow{F}_{M_2\to M_1}$$

$$m_2\overrightarrow{a_2} = \overrightarrow{F}_{ext\to M_2} + \overrightarrow{F}_{M_1\to M_2}$$

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$$\overrightarrow{F}_{M_2\rightarrow M_1} + \overrightarrow{F}_{M_1\rightarrow M_2} = \vec{0}$$

Newton's 3rd law

System comprising multiple parts

If the dynamics of the system S is observed from a Galilean frame of reference, then Newton's 2nd law applies for each part of the system:

$$
? = \quad
\begin{aligned}
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$$

Newton's 3rd law

Reminder on the centre of mass

The centre of mass (CM) of a system S of N points $M_1, \ldots, M_N$ with masses $m_1, \ldots, m_N$ is the geometric point M_{cm} of mass $m_s = m_1 + m_2 + \ldots + m_N$, whose position vector relative to an origin O is given by:

$$\boxed{\vec{r}_{cm} = \frac{1}{m_s} \sum_{i=1}^{N} m_i \vec{r}_i}$$

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Example: For N = 2, $\vec{r}_{cm} = \dfrac{1}{m_1 + m_2}(m_1 \vec{r}_1 + m_2 \vec{r}_2)$

with acceleration: $\overrightarrow{a}_{cm} = \ddot{\vec{r}}_{cm} = \dfrac{1}{m_1 + m_2}(m_1 \ddot{\vec{r}}_1 + m_2 \ddot{\vec{r}}_2)$

$$\longrightarrow \boxed{(m_1 + m_2)\overrightarrow{a}_{cm} = (m_1 \overrightarrow{a}_1 + m_2 \overrightarrow{a}_2)}$$

System comprising multiple parts

If the dynamics of the system S is observed from a Galilean frame of reference, then Newton's 2nd law applies for each part of the system:

$$m_S\,\vec{a}_{cm} = \begin{array}{c} m_1\overrightarrow{a_1} = \overrightarrow{F}_{ext\to M_1} + \overrightarrow{F}_{M_2\to M_1} \\ + \\ m_2\overrightarrow{a_2} = \overrightarrow{F}_{ext\to M_2} + \overrightarrow{F}_{M_1\to M_2} \end{array} \quad = \vec{0}$$

Newton's 3rd law

$$m_S\,\vec{a}_{cm} = \vec{F}_{ext\to M_1} + \vec{F}_{ext\to M_2}$$

System comprising multiple parts

For any extended system S, the dynamics of its centre of mass, as seen in a Galilean frame, is entirely specified by the following equation of motion:

$$\boxed{m_S \, \vec{a}_{cm} = \sum \vec{F}_{ext \to S}}$$

Where the $m_s = m_1 + \ldots + m_n$ is the total mass and $\overrightarrow{a}_{cm}$ the acceleration of the centre of mass.

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Where the $m_s = m_1 + \ldots + m_n$ is the total mass and $\overrightarrow{a}_{cm}$ the acceleration of the centre of mass.

The system S could be stretched or rotating so that the above equation does not fully characterise the dynamics of S.

However, the centre of mass of S is always a good geometrical point to characterise the translation of S via the above equation.

Practical classification of mechanical forces

* ***Forces at a distance (related to "fundamental forces")***
  - Act at any distance from the object they act upon
  - The net point of application of the force on the object is usually the centre of mass
  - Examples: weight, gravitational force, electric force, magnetic force.
  - Can often be determined by formulae or calculations

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  - Examples: weight, gravitational force, electric force, magnetic force.
  - Can often be determined by formulae or calculations
- ***Contact forces***
  - A force that acts only when there is physical contact between two bodies (act solely at the point of contact)
  - Are usually unknown and need supplementary informations to be determined
  - Examples: applied force, reaction force, friction, tension, air resistance etc.

Vertical free fall

A ball of mass m is dropped with initial zero velocity from the top of Lincoln's cathedral whose position vector is $\vec{r}_\text{o} = (83\ \text{m})\,\hat{j}$ in a frame $(O,\ \hat{i},\ \hat{j})$ assumed Galilean.

Question: assuming the air resistance can be neglected, determine the acceleration, velocity and position vectors of the ball.

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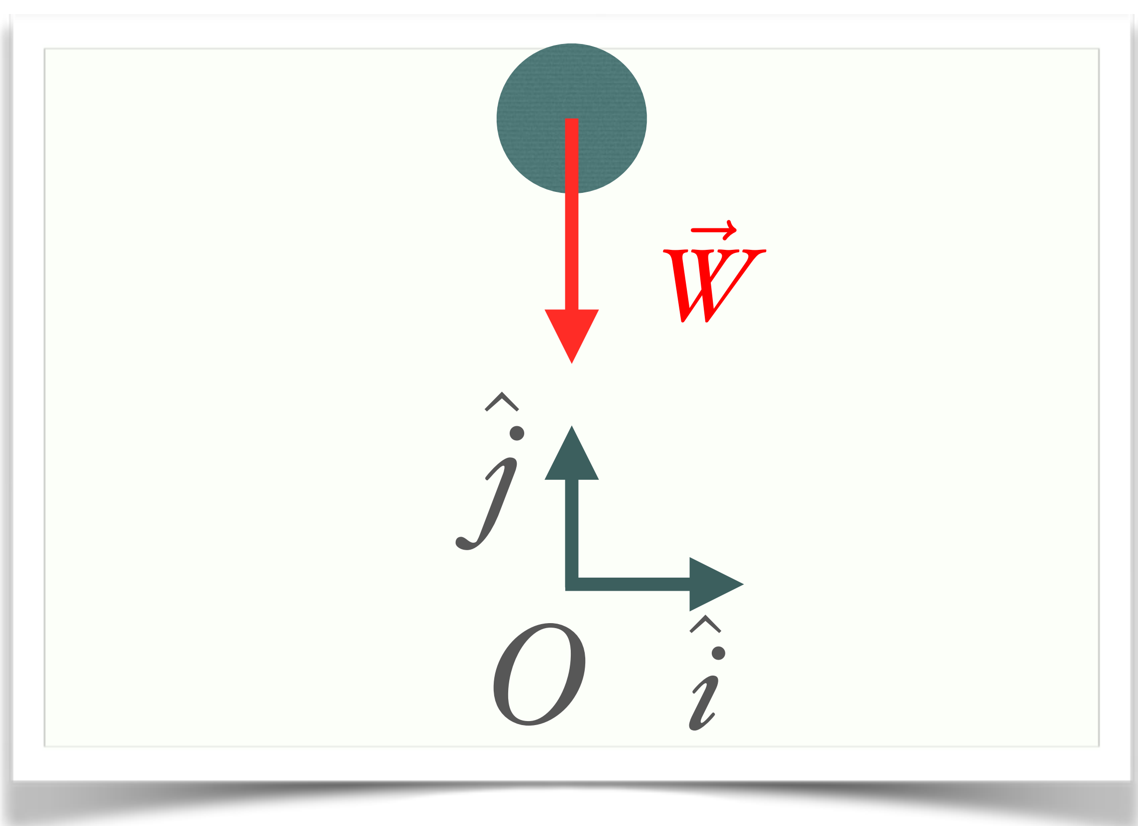
<u>Answer</u>: we shall answer this question in multiple steps.

1st step: make a diagram with all forces



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2nd step: list all the forces acting on the ball

Contact forces	Forces at a distance
None	Weight: $\vec{W} = -m\,g\,\hat{j}$

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3rd step: use Newton's second law

The frame is Galilean therefore Newton's 2nd law applies

$$m\,\vec{a} = \vec{W} = -m\,g\,\hat{j}$$

4th step: get the acceleration vector and components

$$\vec{a} = -g\,\hat{j}\ .\ So,\ a_x = 0,\ a_y = -\,g\ .$$

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<u>Answer</u>: we shall answer this question in multiple steps.

5th step: integrate to get the velocity components and vector

$$\dot{v}_x = a_x = 0,\ so\ v_x(t) = v_x(0) = 0$$

$$\dot{v}_y = a_y = -\,g,\ so\ v_y(t) = v_y(0) - gt = -\,gt$$

$$\boxed{\vec{v} = -g\,t\,\hat{j}}$$

Vertical free fall

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Question: assuming the air resistance can be neglected, determine the acceleration, velocity and position vectors of the ball.

Answer: we shall answer this question in multiple steps.

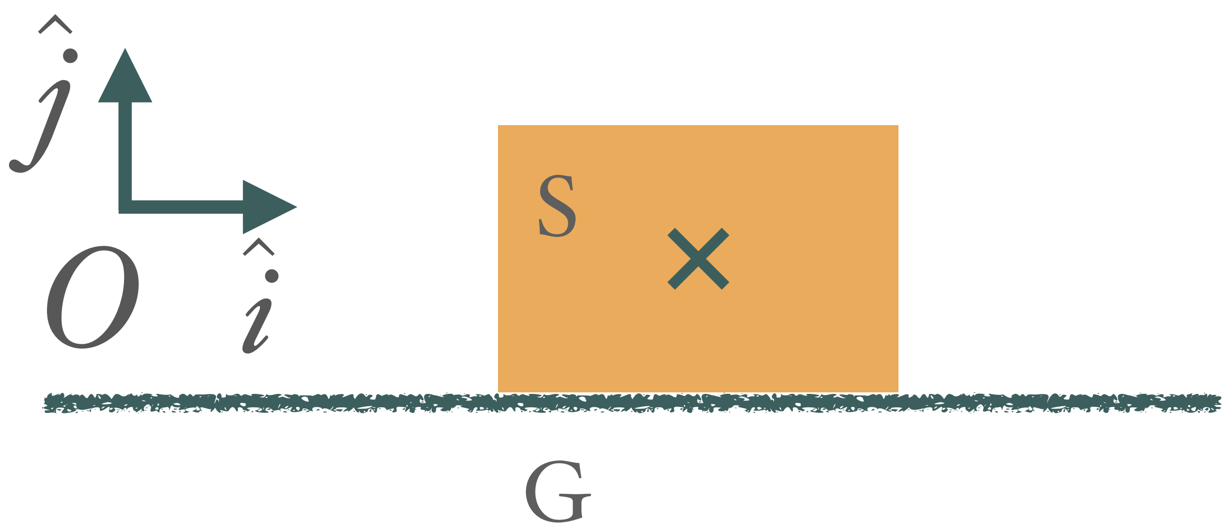
6th step: integrate to get the position vector

$$\dot{x}(t) = v_x(t) = 0,\ so\ x(t) = x(0) = 0$$

$$\dot{y}(t) = v_y(t) = -gt,\ so\ v_y(t) = y(0) - \frac{1}{2}gt^2 = 83m - \frac{1}{2}(9.8\ m\cdot s^{-2})t^2$$

$$\boxed{\vec{r}(t) = [(83\text{ m}) - (4.9\text{ m}\cdot\text{s}^{-2})\,t^2]\ \hat{j}}$$

First encounter with contact forces



A block S of mass m is resting without motion on the ground G in the frame $(O, \hat{i}, \hat{j})$ considered Galilean.

<u>Question</u>: Determine the force exerted by the block on the ground.

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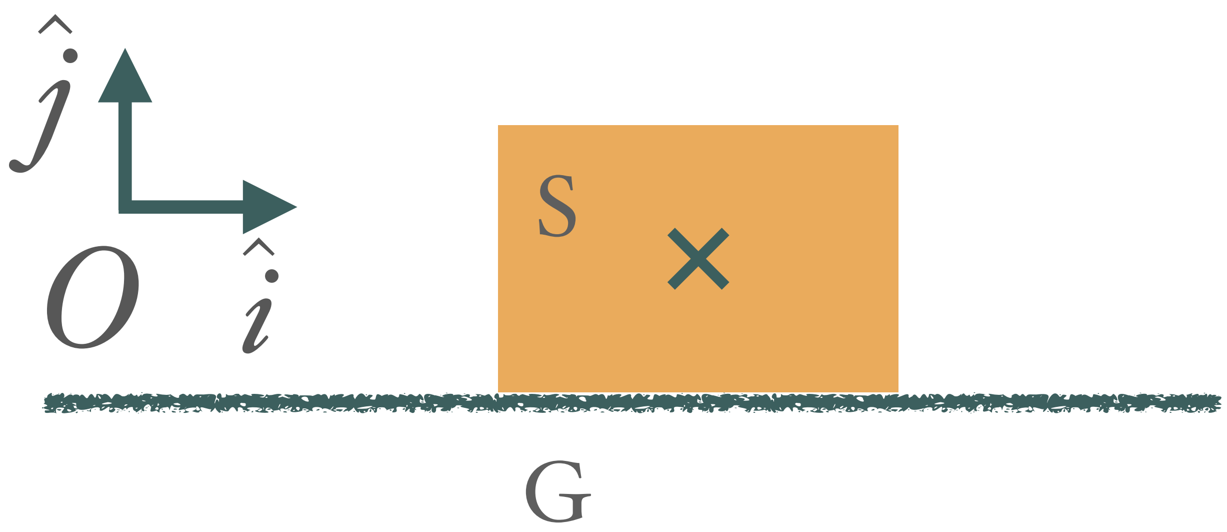

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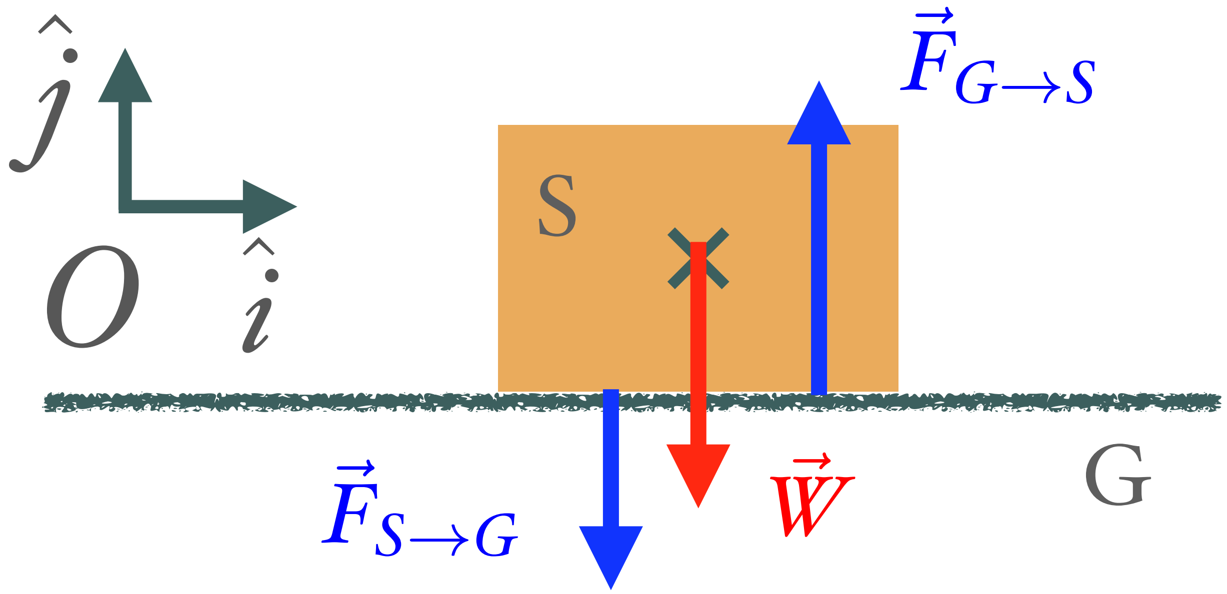

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1st step: make a diagram

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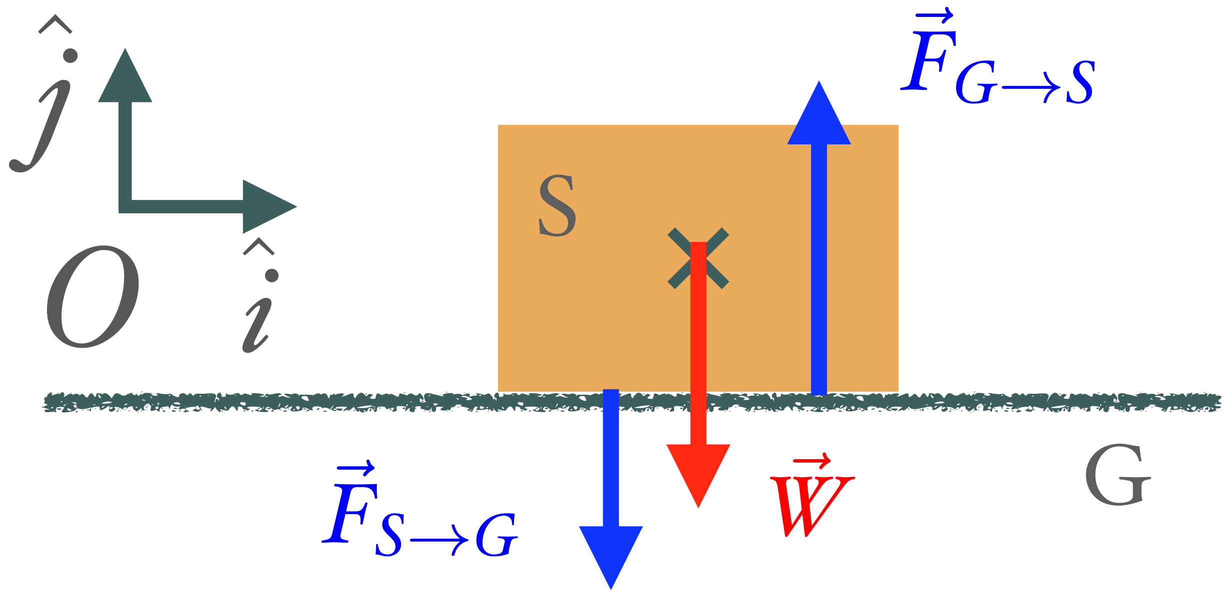

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2nd step: list forces acting on S

Contact forces	Forces at a distance
Ground reaction : $\vec{F}_{G\to S}$	Weight: $\vec{W} = -m\, g\, \hat{j}$

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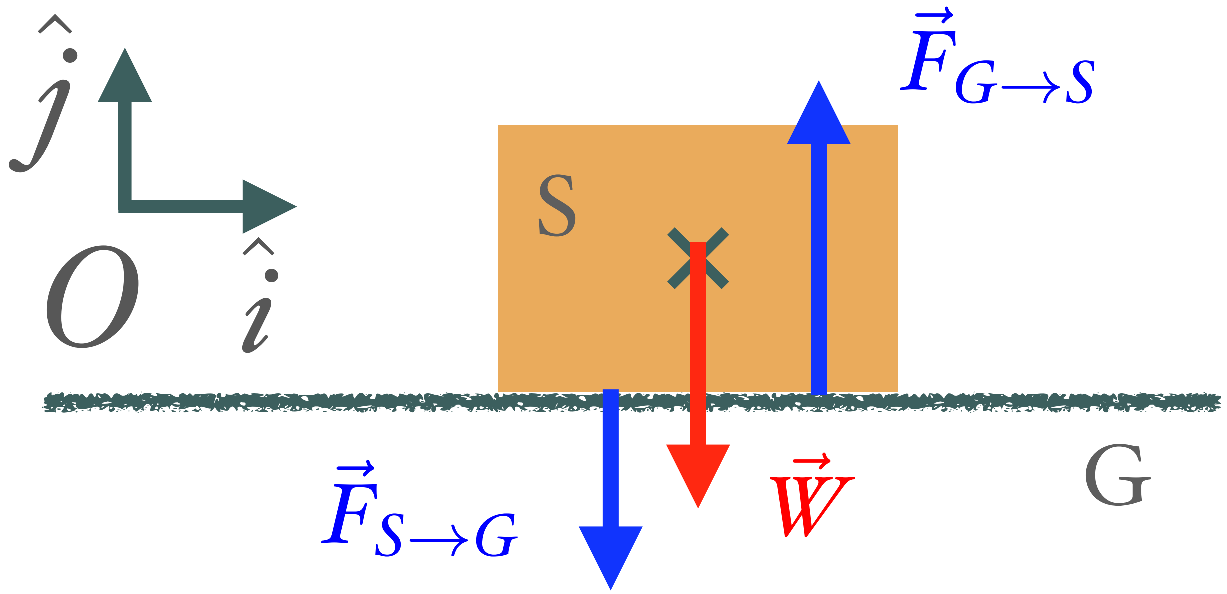

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3rd step: apply Newton's 2nd law

The frame is Galilean therefore Newton's 2nd law applies

$$m\,\vec{a} = \vec{W} + \vec{F}_{G\to S}$$

$$\vec{a} = \vec{0} \implies \vec{F}_{G\to S} = -\vec{W}$$

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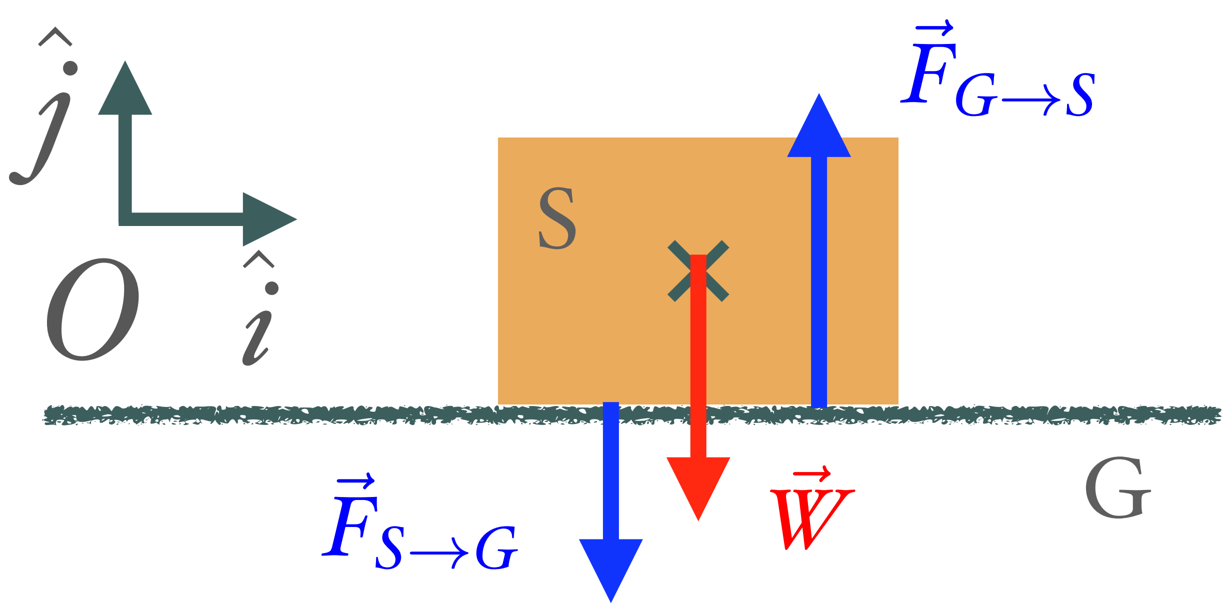

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<u>Answer</u>: Again we proceed in multiple steps

4th step: apply Newton's 3rd law

$$\vec{F}_{S\to G} = -\vec{F}_{G\to S} \implies \boxed{\vec{F}_{S\to G} = \vec{W}}$$