Classical Mechanics

Theodoros Kouloukas

TKouloukas@lincoln.ac.uk

UNIVERSITY OF LINCOLN

What is Classical Mechanics?

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- The physical systems governed by classical mechanics are deterministic, i.e if the present state of the system is known then all the future and past states of the system can be completely determined.

What is Classical Mechanics?

We may regard the present state of the universe as the effect of its past and the cause of its future.

"An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

— *Pierre Simon Laplace, A Philosophical Essay on Probabilities*

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Is nature predictable?

Classical Mechanics

A naive story:

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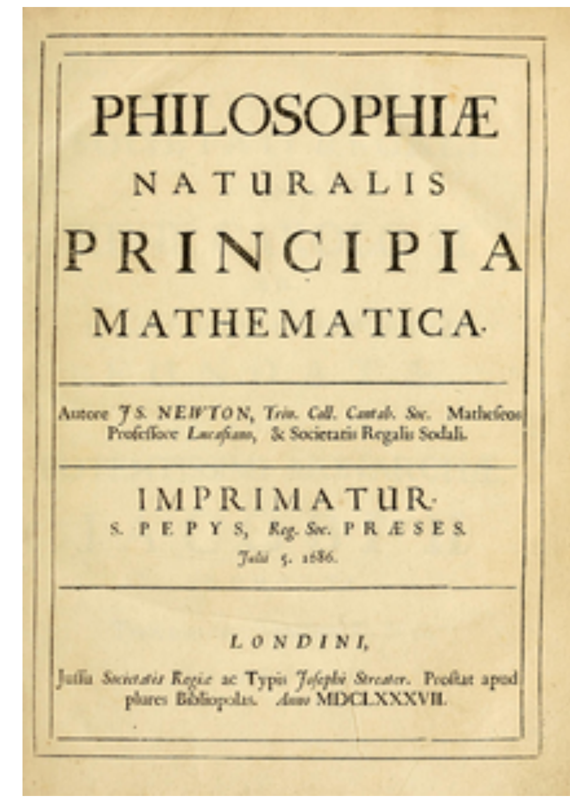

PHILOSOPHIÆ

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Classical Mechanics

A naive story:

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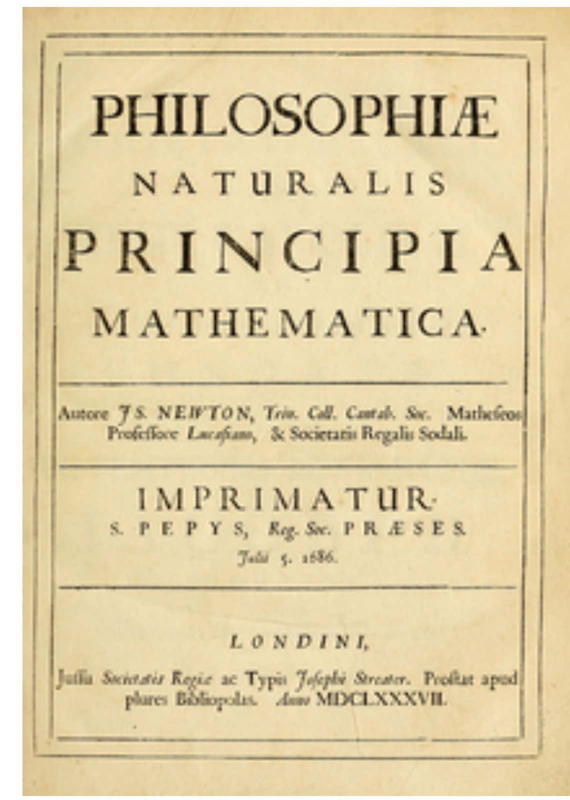

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A more accurate story:

Aristotle, Archimedes, Copernicus, Kepler, Galileo, Huygens, **Newton**, Leibniz, Euler, Lagrange, Laplace, Hamilton, Kovalevskaya, Noether, Kolmogorov,… **and many others!**

The ABCs of Classical Mechanics

Motion

- ***Motion*** is the change of the position of an object in *space* as *time* passes

- The concept of motion, which lies at the heart of classical mechanics, requires the concepts of **space** and **time**.

- In classical mechanics, space and time are ***primitive*** concepts: their meaning is considered self-evident and they do not require a definition.

Not requiring a definition of space and time does not mean that they are easy to apprehend.

The two pillars of classical mechanics

Classical mechanics is supported by two complementary sub-disciplines: *kinematics* and *dynamics*

Kinematics describes the motion of objects without considering the forces that cause them to move (*description of motion*).

Dynamics concerns the study of forces and their effects on motion (*explanation of motion*).

Introduction to Kinematics

Describing position, time and motion

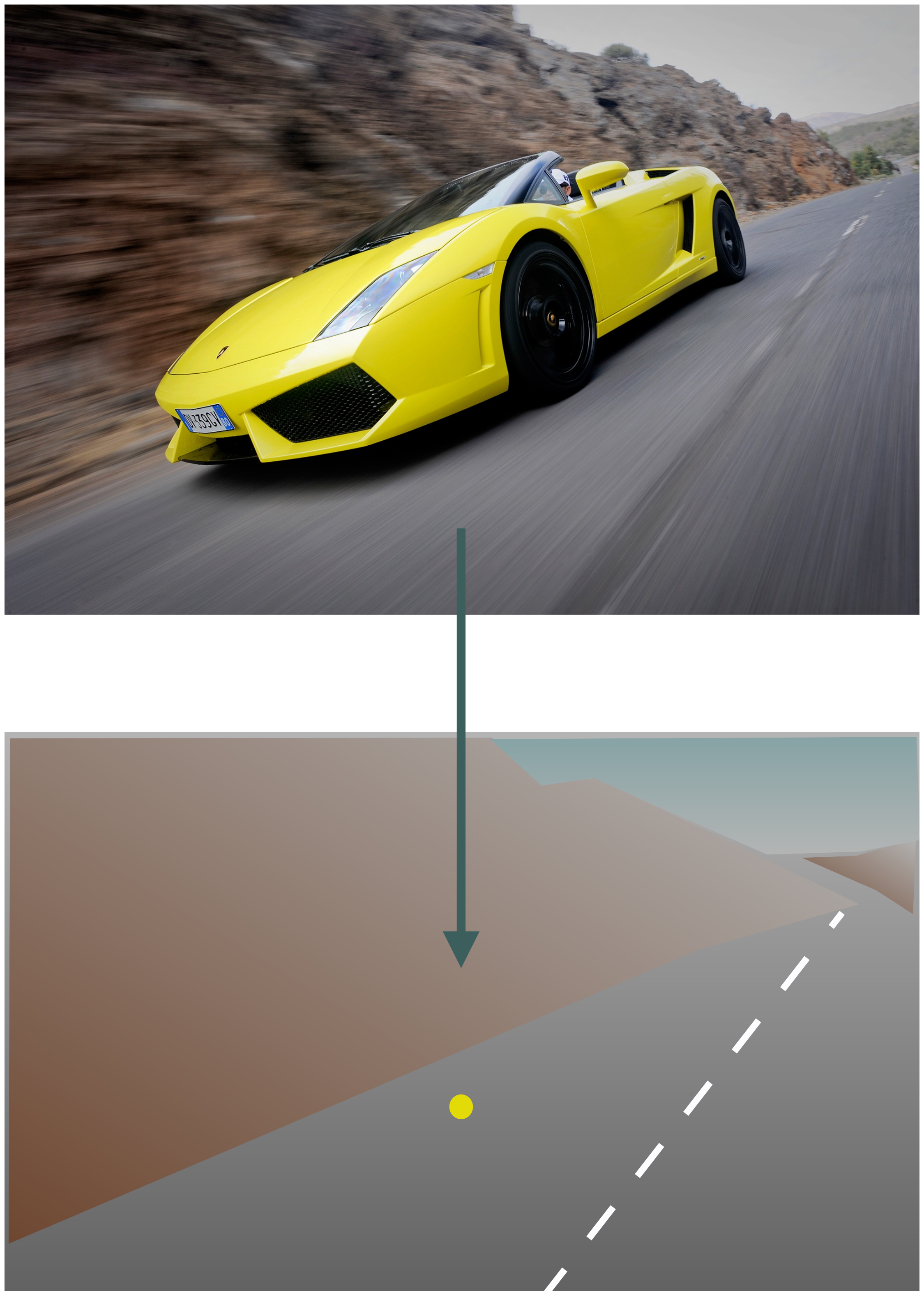
The path to abstraction

Not all details matter to characterise the motion of an “object”:

- Details **unrelated** to the description of motion like car colour, brand of the car, etc…

- Details **contributing** to motion but either too complex to be modelled or can be abstracted away for the motion under study; like pistons in the car engine, exhaust gases, shape of the car etc…

The path to abstraction



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→ **Point Object**

Trajectory

Once an object is abstracted as a point object it is often denoted by an upper case letter of the latin alphabet (A, B, C, etc…) representing the object.

For example, we could have (but it's up to your imagination!):

- Car → point C
- Plane → point P
- Earth → point E
- Sun → point S

Trajectory

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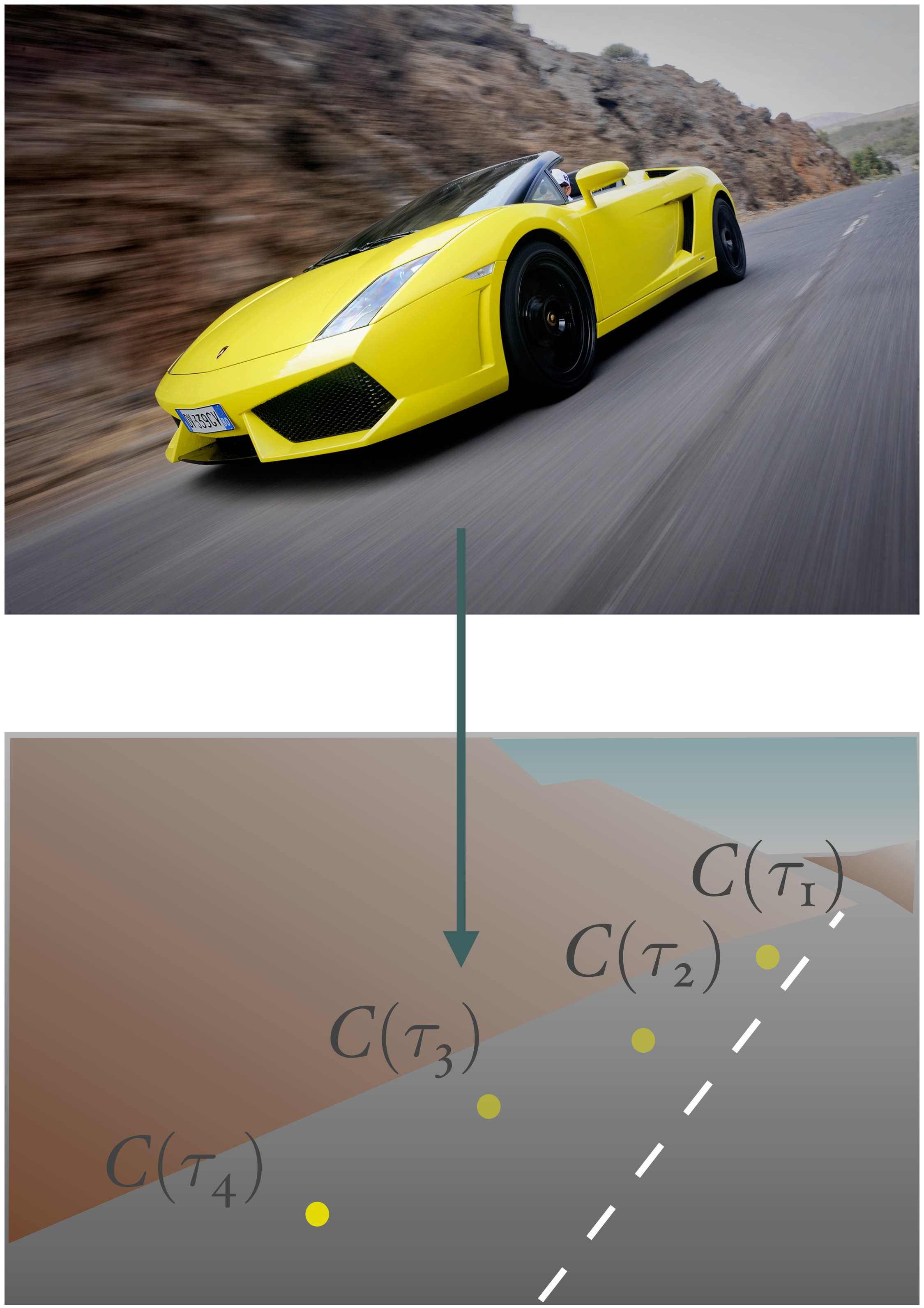
- Car $\longrightarrow$ point C
- Plane $\longrightarrow$ point P
- Earth $\longrightarrow$ point E
- Sun $\longrightarrow$ point S

The succession of different places a point object occupies at different instants sorted in an ordered sequence $(\tau_1, \tau_2, \ldots)$ is simply denoted $(C(\tau_1), C(\tau_2), \ldots)$ and called the ***trajectory*** of the (point) object.

Trajectory



For example, in the case of the racing car, the point C representing the car occupies successively the positions

$$C(\tau_1), C(\tau_2), C(\tau_3) \text{ and } C(\tau_4)$$

Relative position in one dimension and time intervals

From place to (relative) position

If you were to live on an infinite straight line how would you answer a question like: “where are you?”

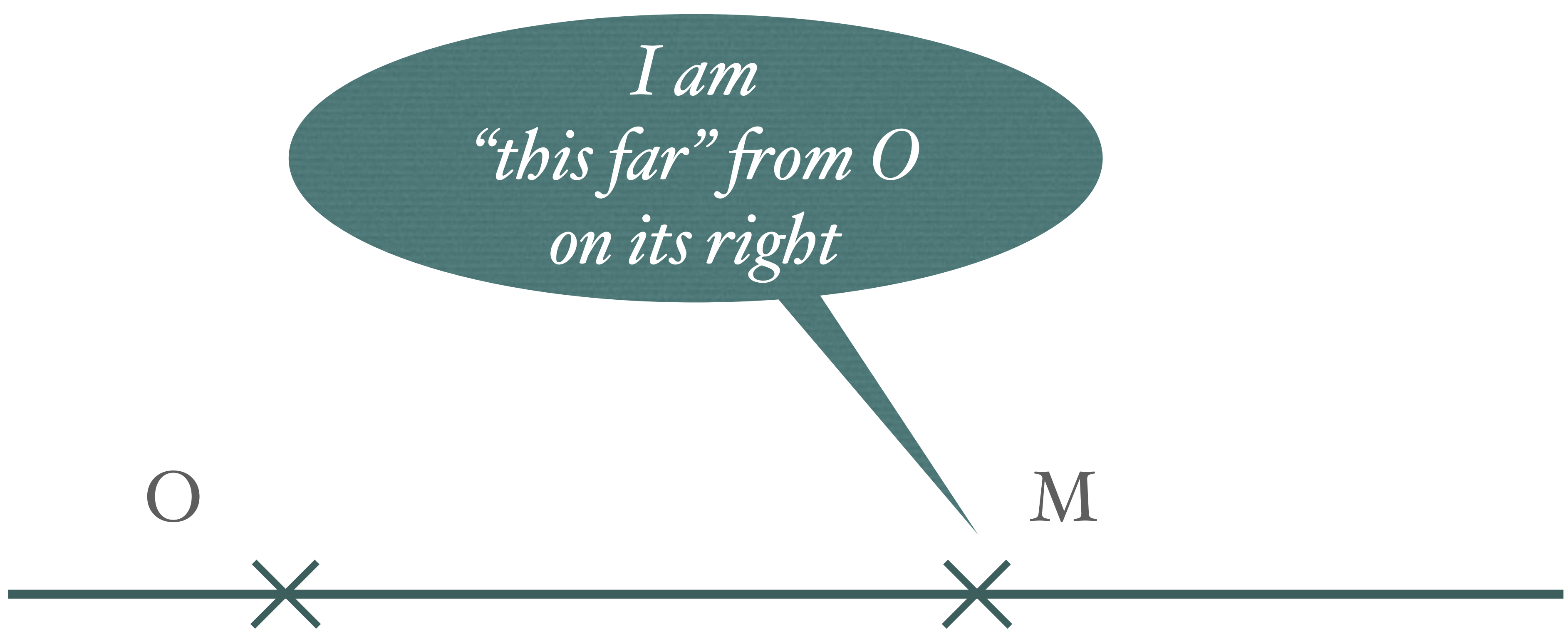
From place to (relative) position

If there is nothing around you, the most reasonable answer you can give is likely to be:

...but that would not tell much

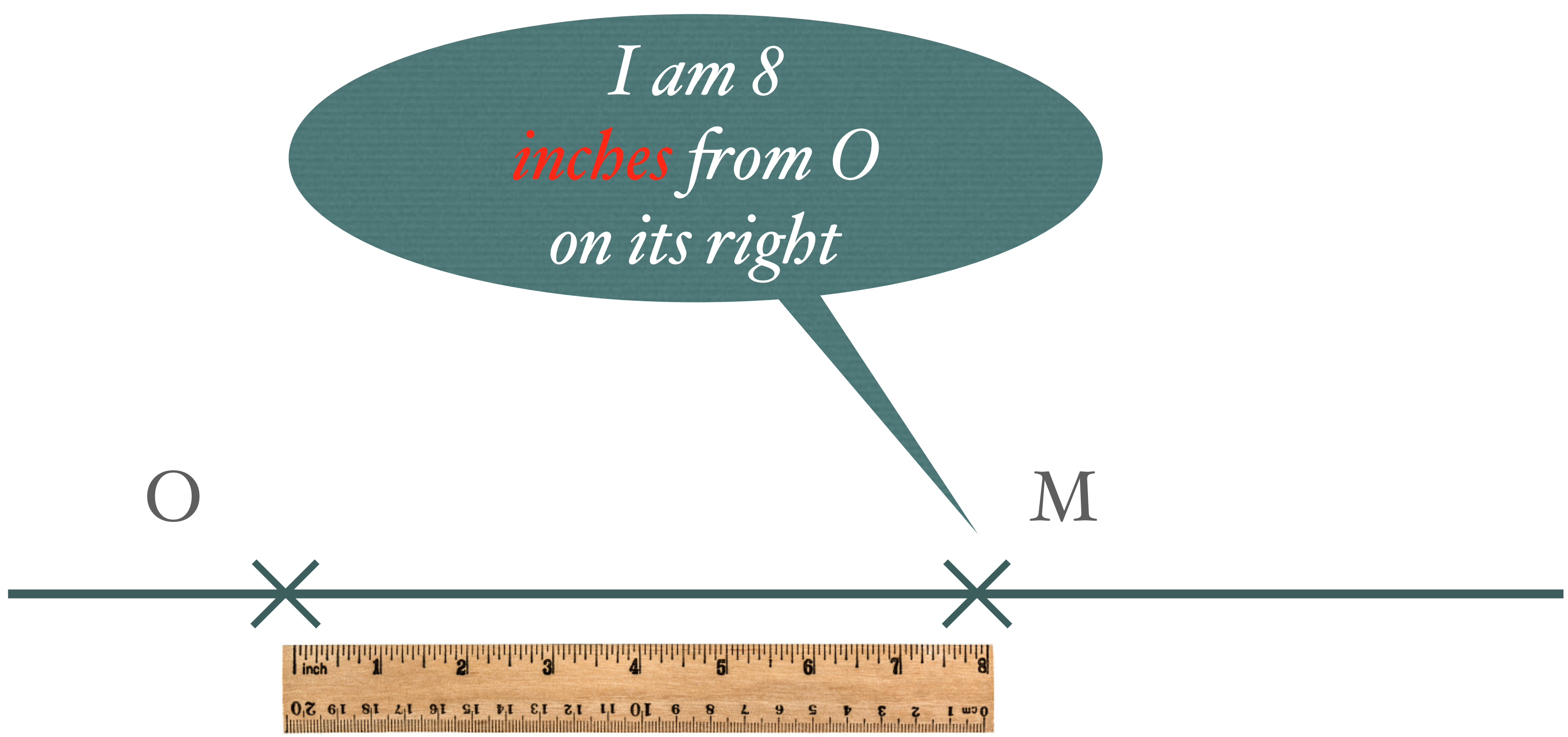
From place to (relative) position

If there is something around you, you could express your current location *relative* to another object:



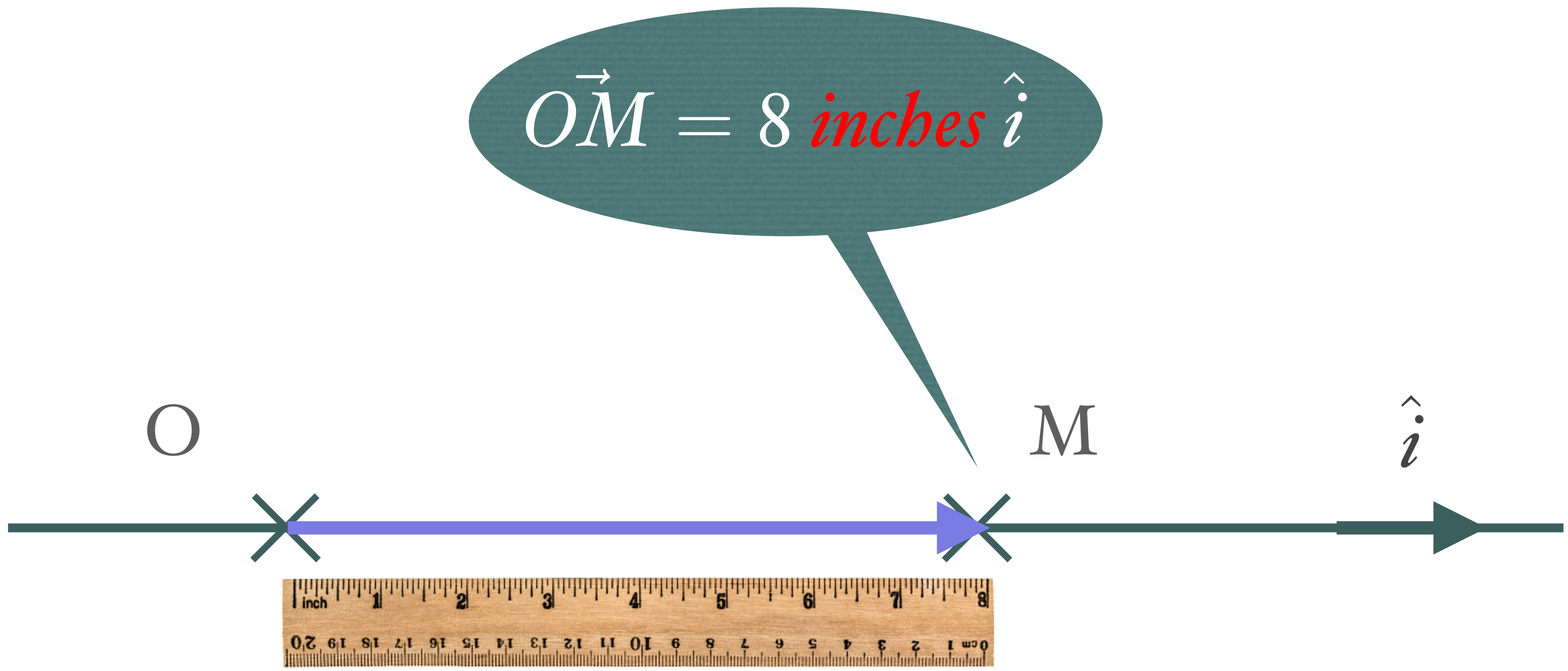
From place to (relative) position

If moreover you are given a standard of measure for lengths, i.e. a unit of length (such as inch, m, cm, feet, etc.) you could express your *relative position* more quantitatively:



From place to (relative) position

By choosing an orientation $\hat{i}$ to the line, you can state your relative position more formally as a ***position vector***:



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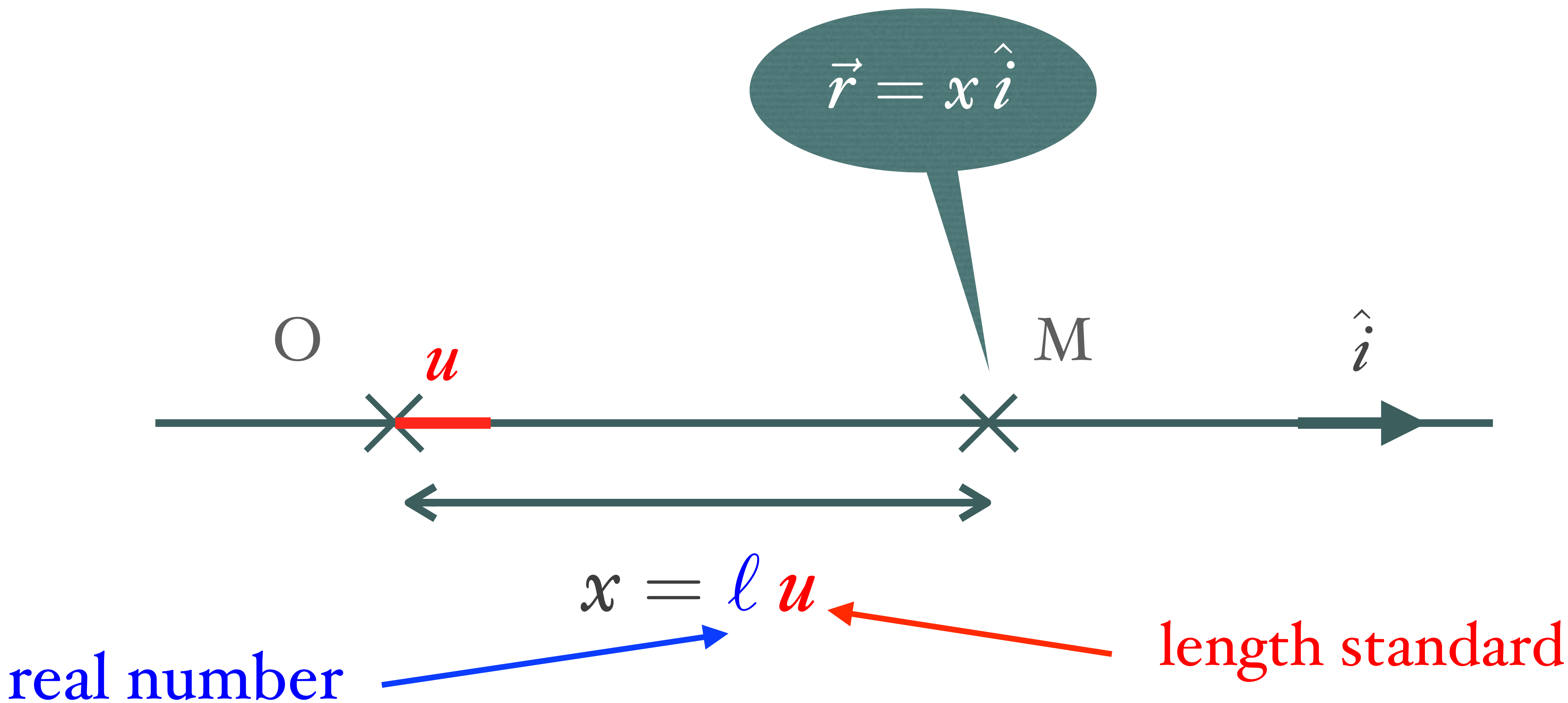

From instants to time intervals

What if you were asked the simple question "what instant is it?"
Is there an absolute way to answer to it?

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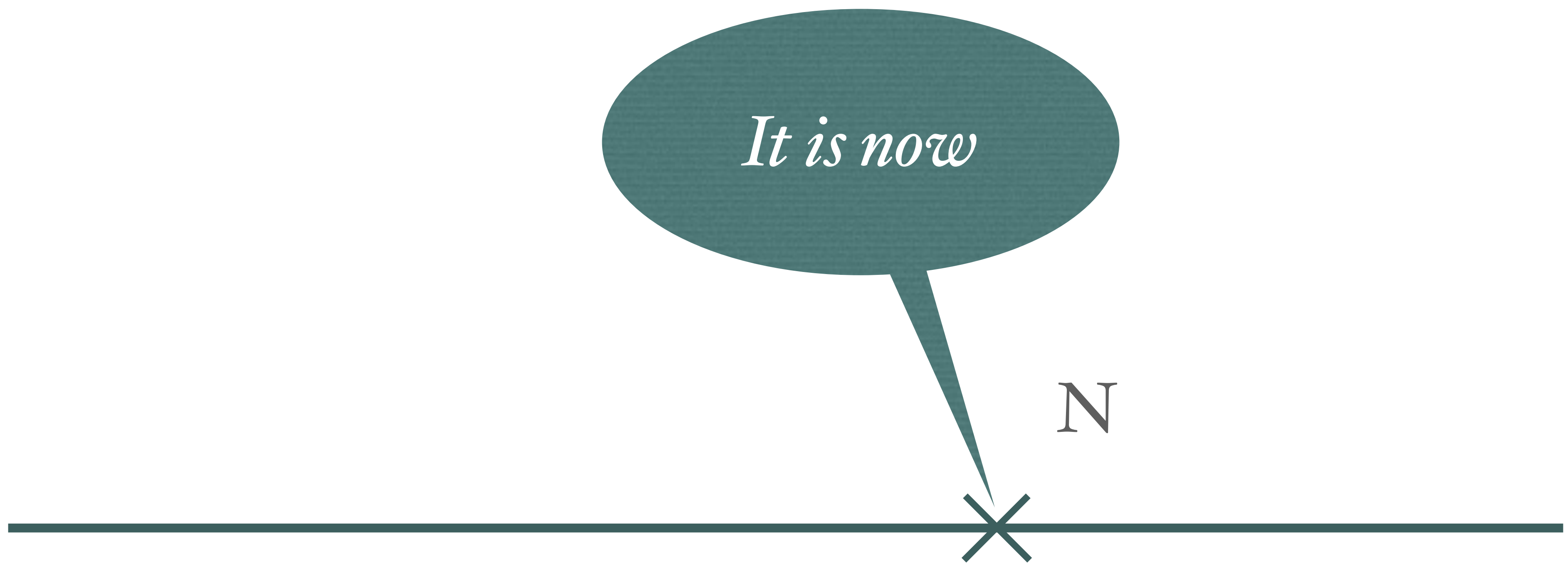
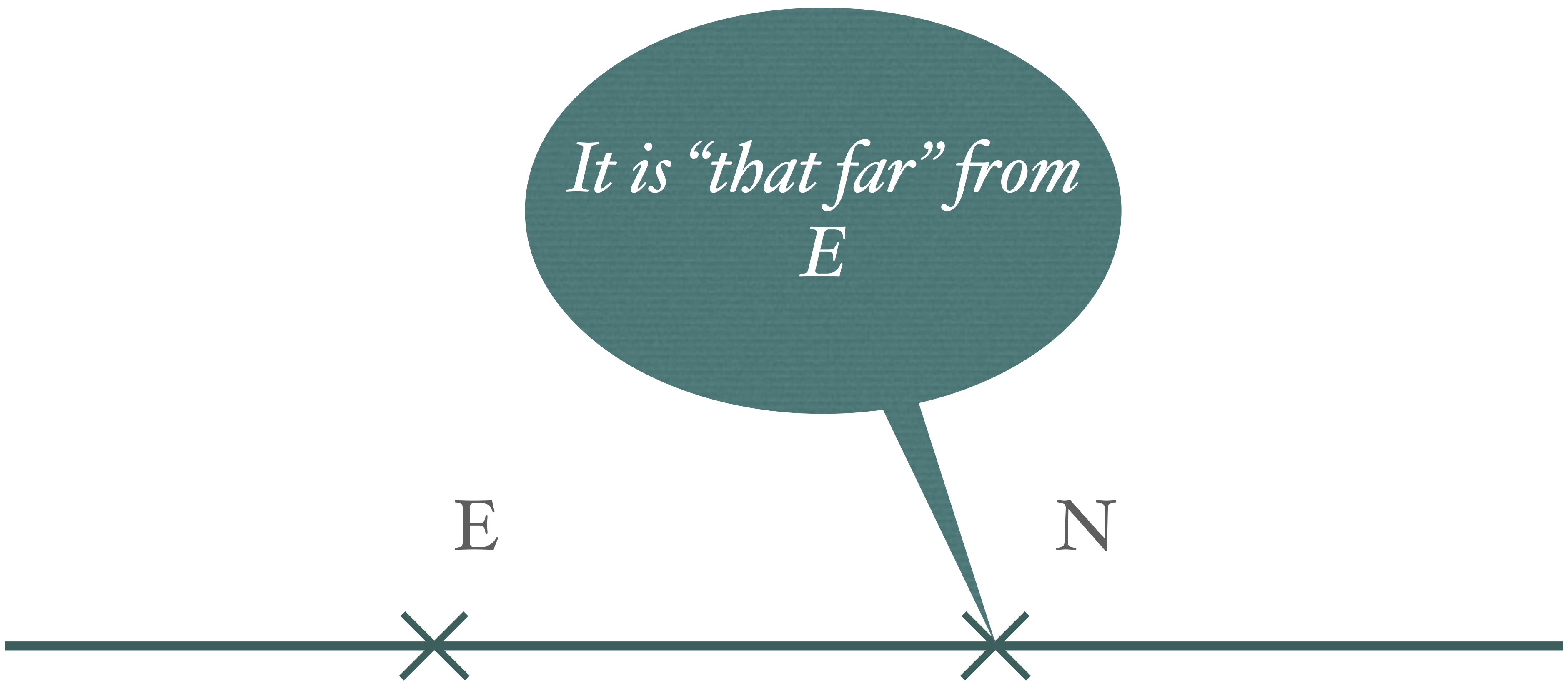


perfectly correct but sort of useless

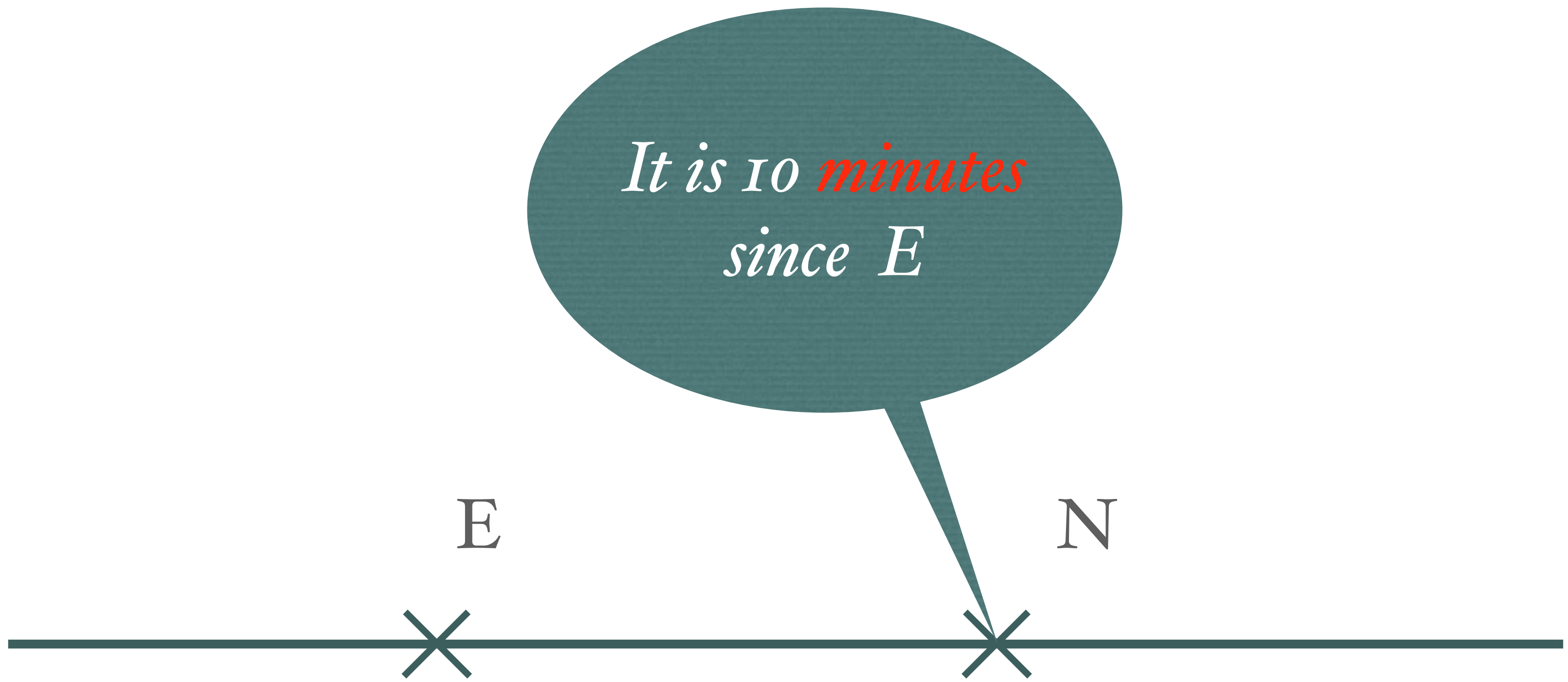
From instants to time intervals

If you have an event E of reference then you could say



From instants to time intervals

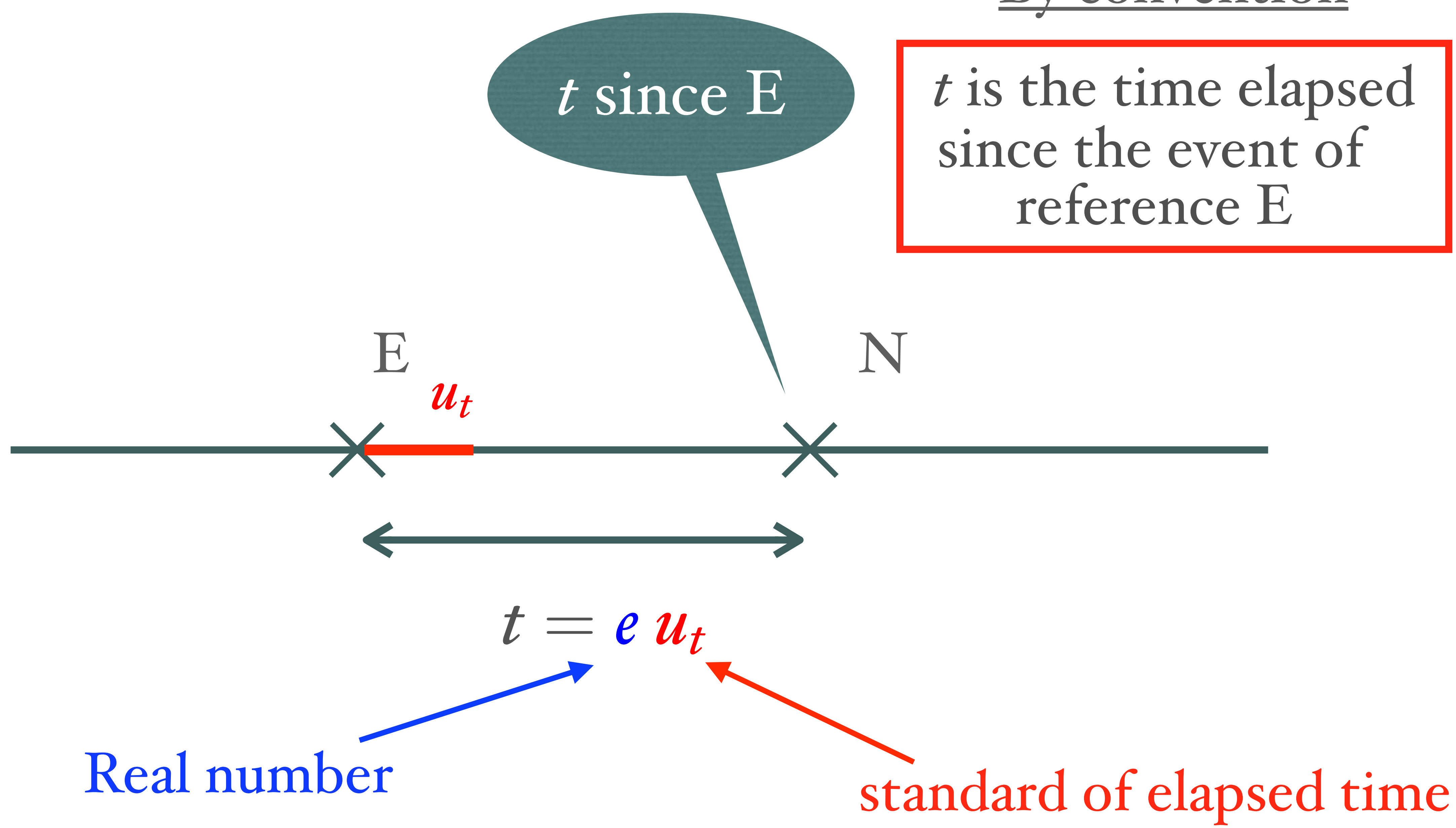
The “that far” must be measured with a standard for measuring the time elapsed between two events, i.e. a unit of time such as hours, seconds, minutes, years)



From instants to time intervals

By convention

t is the time elapsed since the event of reference E



Introduction to ‘physical dimension’

Physical Dimensions

We have seen that $x = \ell u \; and \; t = e u_t$, where

- x denotes a relative position in a line and ℓ a real number related to a unit of length u (such as inch, m, km, feet)
- t denotes the time elapsed since an event of reference and e a real number related to a unit of time u_t (seconds, minutes, years)

Physical Dimensions

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> In physics the word ***dimension*** denotes the physical nature of a quantity (represented by a given symbol).

1. The dimension of x, denoted $[x]$, is length and we write $[x] = L$
2. The dimension of t, denoted [t], is time and we write $[t] = T$

Similarly, we denote $[u] = L$ and $[u_t] = T$.

The real numbers ℓ and e are considered *dimensionless* and we write $[\ell] = [e] = 1$.

Dimensional Analysis

Dimensions can be treated as algebraic quantities.

Basic rules:

- Quantities can be added or subtracted only if they have the same dimensions and $[a+b] = [a] = [b]$ if $[a] = [b]$
- Numbers are dimensionless: $[a] = 1$, for any number a
- The dimension of the product of two quantities is the product of their dimensions, i.e. $[ab] = [a][b]$
- $[a^p] = [a]^p$ for any rational number p

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Example If x denotes a relative position in $1D$ and t a time interval then

$$[\frac{2x}{t}]=\frac{[2][x]}{[t]}=\frac{1\cdot L}{T}=\frac{L}{T}$$

Notion of complete equation

A general (physics) equation reads:

Left hand side = Right hand side

<u>Definition</u>: An equation is said to be a ***Complete Equation*** if the dimension of its left hand side is equal to the dimension of its right hand side. The numerical values of both sides of a complete equation must be equal in all units that are multiple of the original units.

<u>Examples</u>

- The equation $x = t$ is not a complete equation
- The equation $x = (1km/h)t$ is a complete equation

Notion of complete equation

Physics theories preferentially use complete equations as they allow to relate physical concepts with each other and not just their numerical values in some units; in other words, complete equations carry **“meaning”** .

This is to be opposed to the very useful but less meaningful concept of **empirical equations** which are meant to work for practical situations but only in certain specific units.

Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another or convert within a system (for example, from kilometers to meters).

- The **SI** (*International System of Units*) unit of length is the metre (m) and of time is the second (s).

Example

$$2km/h = 2\frac{1000m}{3600s} = 0.555m/s$$

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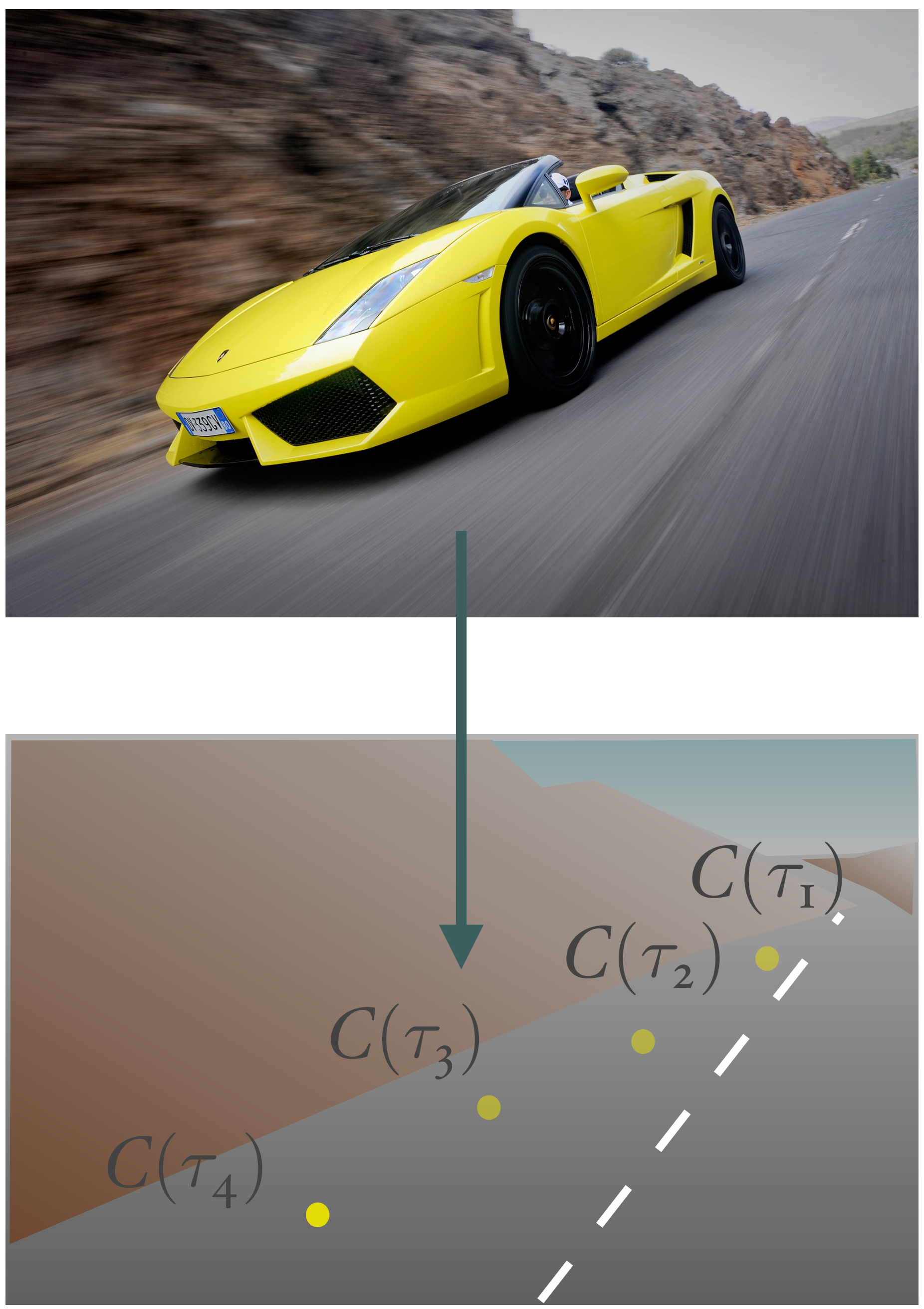
- Like dimensions, units can be treated as algebraic quantities that can cancel each other. By including the units in every step of a calculation, we can detect errors if the units for the answer turn out to be incorrect.

Kinematics in one linear dimension: tools for describing motion

From trajectory to equation

We have seen that a trajectory was a succession of places



$$C(\tau_1),\ C(\tau_2),\ C(\tau_3) \text{ and } C(\tau_4)$$

Given a point of origin they can be related to the vector positions

$$\vec{r}(\tau_1),\ \vec{r}(\tau_2),\ \vec{r}(\tau_3) \text{ and } \vec{r}(\tau_4)$$

Given a direction $\hat{i}$ in 1D, it simplifies even further into

$$x(\tau_1), x(\tau_2), x(\tau_3) \text{ and } x(\tau_4)$$

Given an instant of reference, we get

$$x(t_1), x(t_2), x(t_3) \text{ and } x(t_4)$$

From trajectory to equation

In general we can then characterise the trajectory with a mathematical *function* $x(t)$ such that when evaluated at individual time intervals it gives back the values $x(t_1), x(t_2), x(t_3)$ and $x(t_4)$

$x(t)$ is called the (explicit) ***equation of motion*** of the system and it is often assumed that this function is at least *continuous* and *differentiable* (this is where proficiency in calculus becomes useful).

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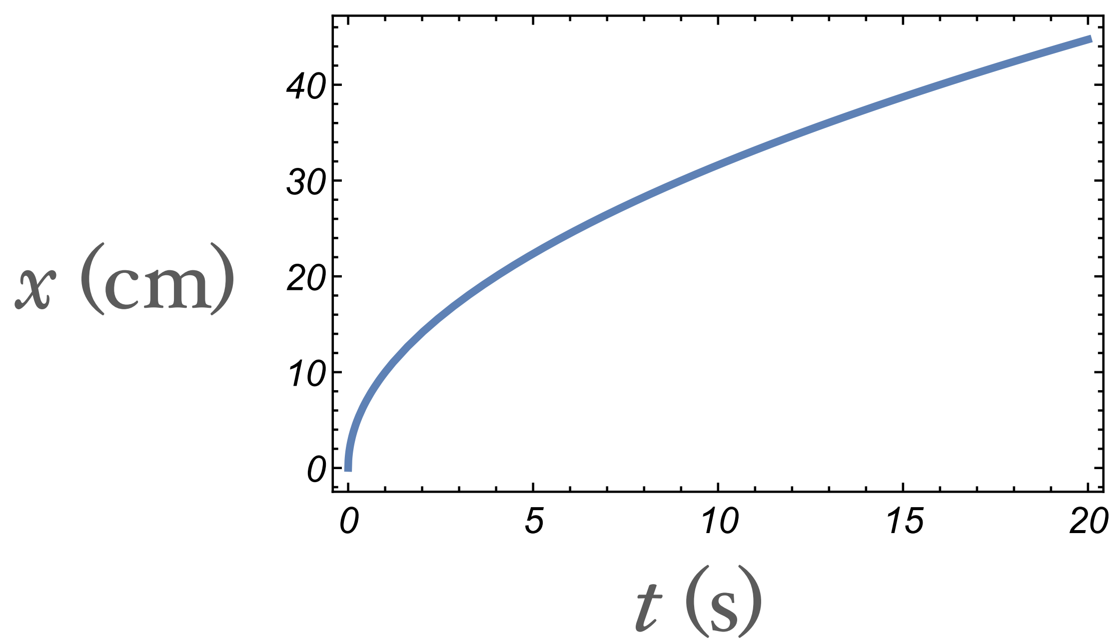
Example 1

Equation of motion

$$x(t) = 10\ cm \cdot s^{-1/2}\sqrt{t}$$

Graph

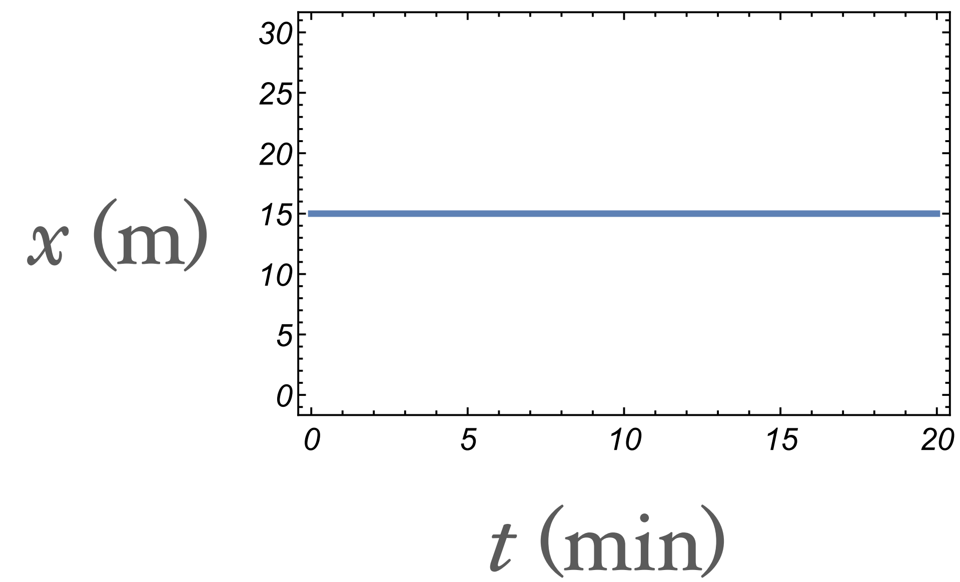


From trajectory to equation

Example 2

$x(t) = 15\ \mathbf{m}$

Graph



From trajectory to equation

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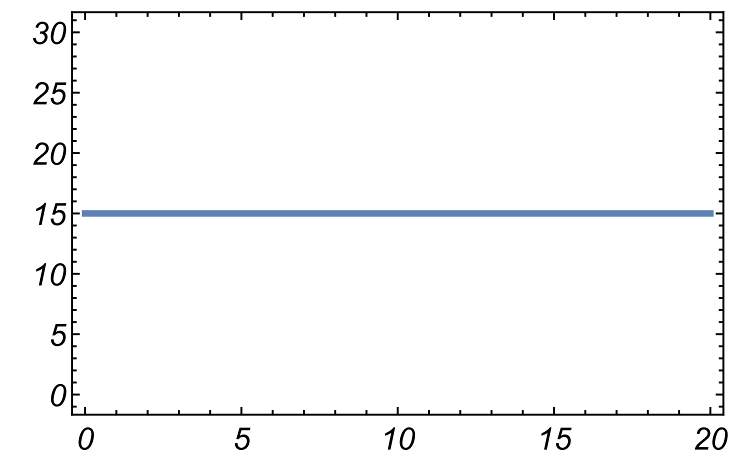
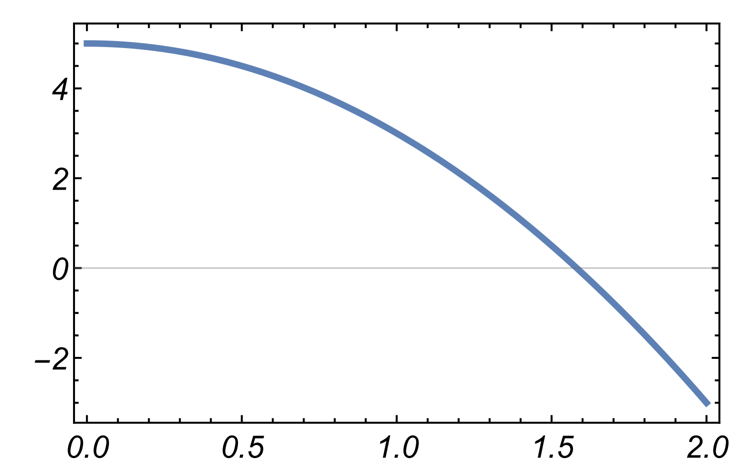


Example 3

$x(t) = (5\ \text{m}) - (2\ \text{m.h}^{-2})t^2$



Velocity

Average velocity and speed

- The ***average velocity*** $v_{x,avg}$ of an object between two times t_1 and t_2 is defined as the average rate of change of $x(t)$:

$$v_{x,avg} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

- The ***average speed*** v_{avg} of an object is defined as **the total distance** d traveled divided by the total time interval $t_2 - t_1$ required to travel that distance:

$$v_{avg} = \frac{d}{t_2 - t_1}$$

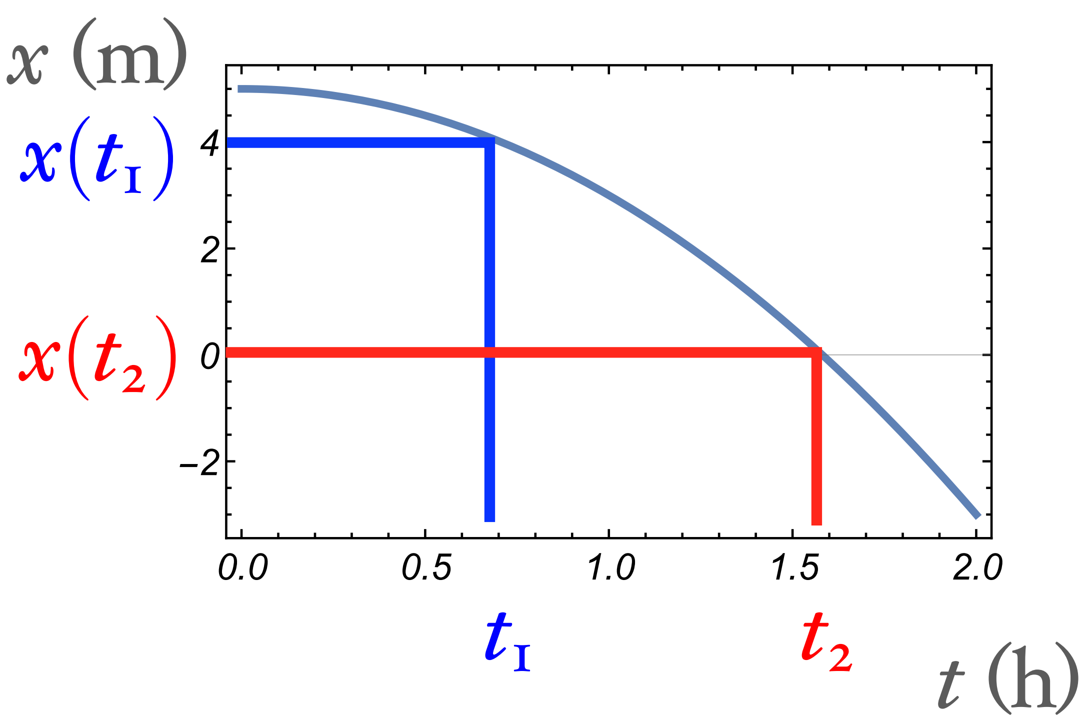
The average velocity and speed have dimensions of length divided by time (L/T) and meters per second in SI units.

Average velocity and speed

Example

The position of a particle moving on x axis varies with time according to the expression $x(t) = (5\ \mathrm{m}) - (2\ \mathrm{m.h^{-2}})t^2$. Find the average velocity and speed between $t_1 = 0.7\mathrm{h}$ and $t_2 = 1.55\mathrm{h}$.

$$v_{x,avg} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$ and $$v_{avg} = \frac{d}{t_2 - t_1}$$ with $x(t_1) = 4.02m,\ x(t_2) = 0.195m$



Average velocity and speed

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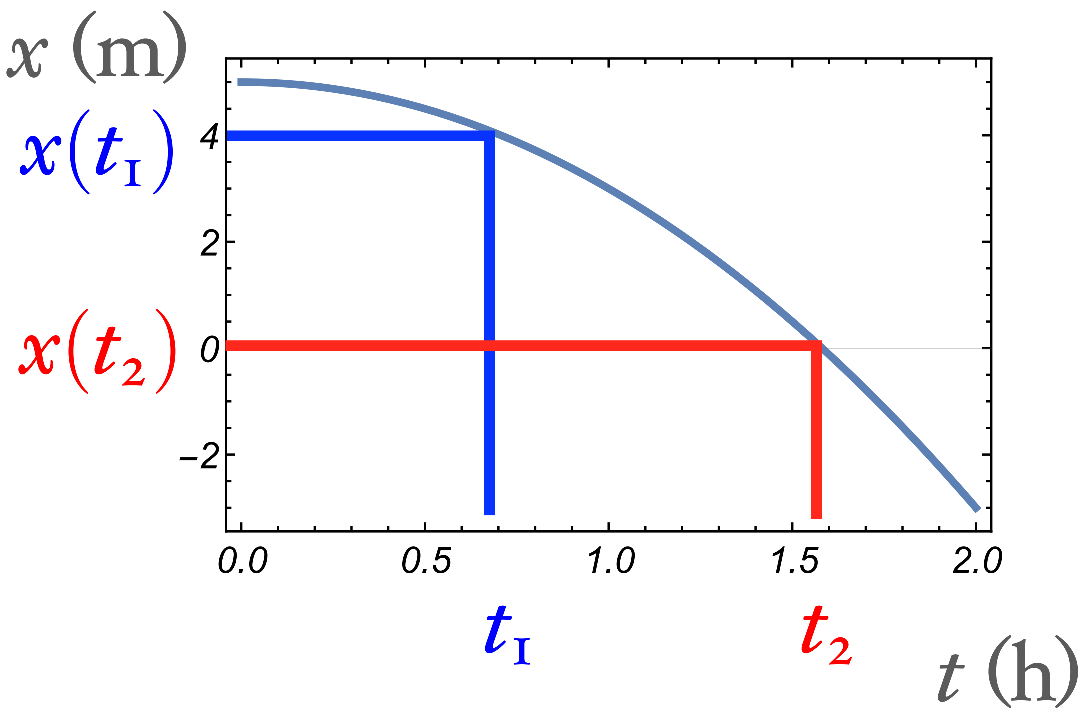
$$v_{x,avg} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} \text{ and } v_{avg} = \frac{d}{t_2 - t_1} \text{ with } x(t_1) = 4.02m,\ x(t_2) = 0.195m$$



$$v_{x,avg} = \frac{(0.195 - 4.02)m}{(1.55 - 0.7)h} = -4.5m/h$$

$$v_{avg} = \frac{(|0.195 - 4.02|)m}{(1.55 - 0.7)h} = 4.5m/h$$

Reminder on calculus: definitions

We consider below functions of a single variable that take a real number as an argument and return a real number. We assume the concept of limit to be known.

Definition 1: a function $f(x)$ in $\mathbb{R}$ is said to be ***continuous*** on an interval $I = [x_{min}, x_{max}]$ if for any a in I it satisfies $\lim_{x \to a} f(x) = f(a)$

Definition 2: a continuous function is said to be ***differentiable*** at point a if the limit $\lim_{h \to 0} [f(a+h) - f(a)]/h$ exists

Definition 3: if a function f is differentiable at point a then one defines $f'(a)$ the ***derivative*** of f at point a as:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Reminder on calculus: derivatives

Functions	Derivatives
Cx^{α}	$\alpha C x^{\alpha - 1}$
Ce^{x}	Ce^{x}
$\ln x$	$\frac{1}{x}$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$

The "dot" notation

In mechanics, time derivatives play a central role both in the description of motion and in its prediction. On another hand modern mechanics also uses spatial derivatives for another purpose; it is thus usual to use a dedicated notation to talk about time derivatives: that's the "dot" notation.

Let us consider a function $f(t)$ continuous and differentiable.

We then denote $\dot{f}(t)$ its time derivative at time t such that

$$\dot{f}(t) \equiv \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} = \frac{df(t)}{dt}$$

multiple time derivatives are simply denoted by adding dots

for example $$\frac{d^3 f(t)}{dt^3} = \dddot{f}(t)$$

Velocity and speed

Sometimes, being able to determine the average velocity between two times that are apart is not indicative enough, we would like some quantity associated to a given time

Definition: the ***instantaneous velocity*** at time t_1 is the limit of the average velocity as t_2 tends to t_1:

$$v_x(t_1) = \lim_{t_2 \to t_1} \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{dx(t_1)}{dt}$$

In 'dot' notation: $v_x(t_1) = \dot{x}(t_1)$

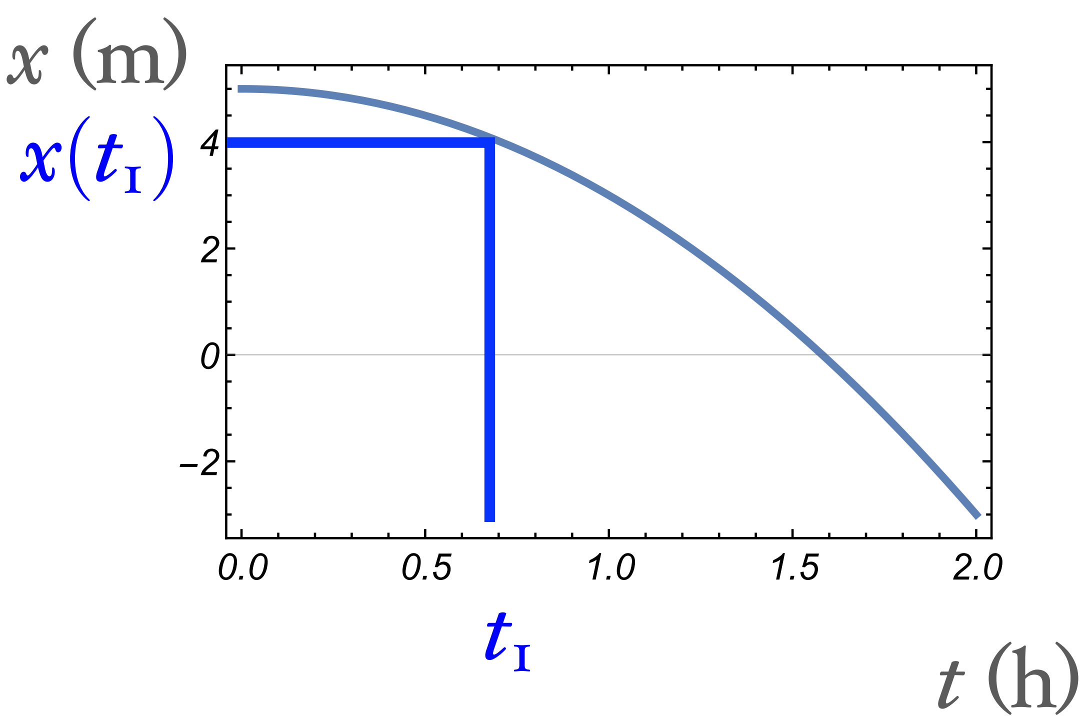
Definition: We call the ***instantaneous speed*** at time t_1, the magnitude of the instantaneous velocity at that time:

$$v(t_1) = |v_x(t_1)|$$

Velocity and speed

Example

The position of a particle moving on x axis varies with time according to the expression $x(t) = (5\ \text{m}) - (2\ \text{m.h}^{-2})t^2$. Find the instantaneous velocity and speed at $t_1 = 0.7\text{h}$.



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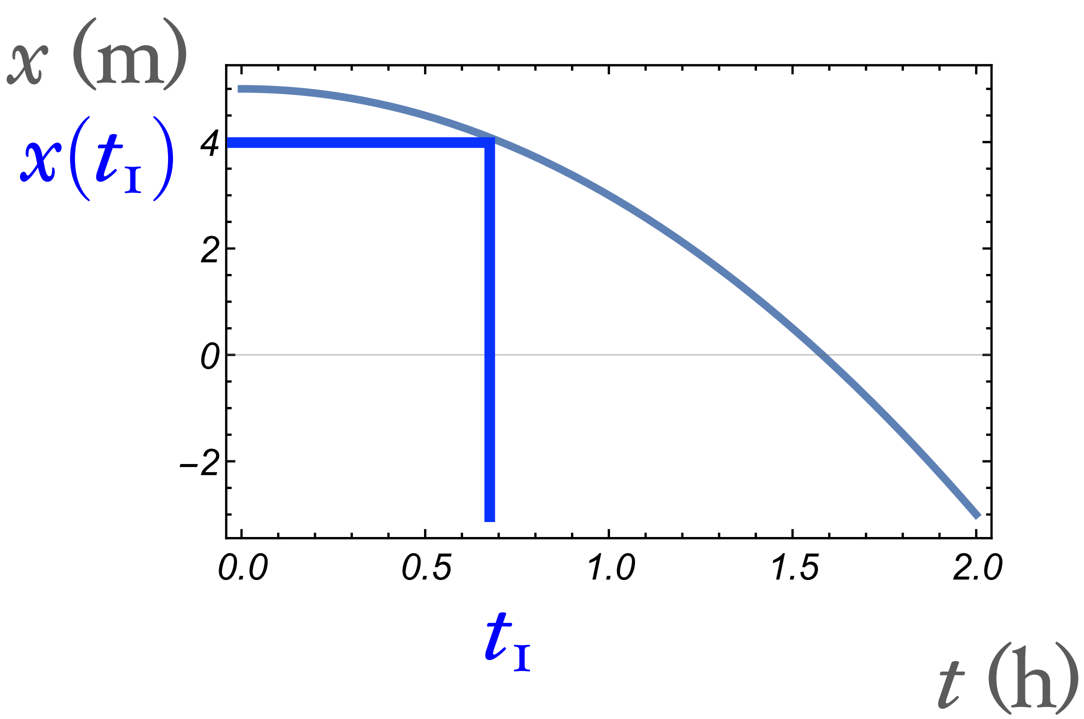
$$v_x(t) = \frac{dx(t)}{dt} = (-4\ m \cdot h^{-2})t$$



$$v_x(0.7h) = -\ 2.8\ m/h$$

$$v(0.7h) = |-2.8|\ m/h = 2.8\ m/h$$

Acceleration

Acceleration

The velocity informs about the rate of change of the relative position. But an object initially at rest does not get to a finite velocity in the blink of an eye; it needs to accelerate...

- We define the ***average acceleration*** $a_{x,avg}$ of an object between two times t_1 and t_2 as the rate of change of $v_x(t)$

$$a_{x,avg} = \frac{v_x(t_2) - v_x(t_1)}{t_2 - t_1}$$

- We define the ***instantaneous acceleration*** at t_1 as being the limit of the average acceleration as the time t_2 tends to t_1

$$a_x(t_1) = \lim_{t_2 \to t_1} \frac{v_x(t_2) - v_x(t_1)}{t_2 - t_1} = \dot{v}_x(t_1) = \ddot{x}(t_1)$$

Acceleration

Example

The position of a particle moving on x axis varies with time according to the expression $x(t) = 10\ m \cdot s^{-1/2}\sqrt{t}$.

i) Express the velocity and the acceleration as functions of time and evaluate the acceleration at $t_1 = 5s$.

ii) Find the average acceleration between $t_1 = 5s$ and $t_2 = 15s$.

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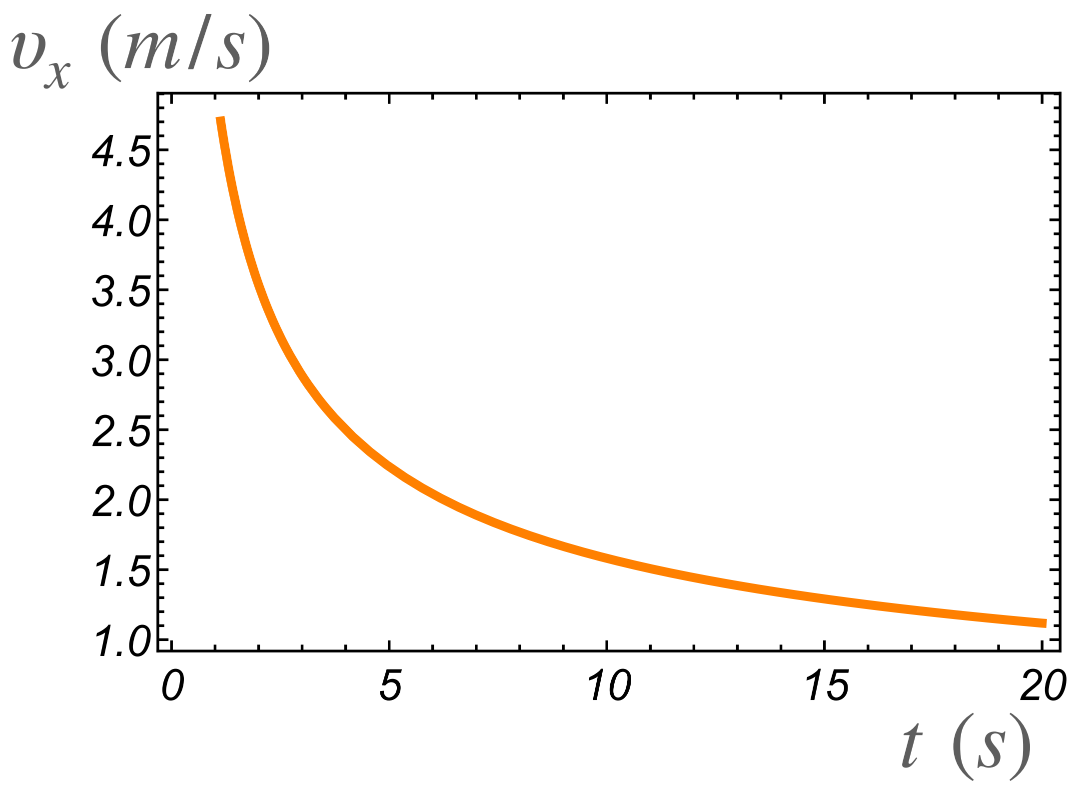

$$v_x(t) = \dot{x}(t) = \frac{(5\ m \cdot s^{-1/2})}{\sqrt{t}}$$

$$a_x(t) = \dot{v}_x(t) = (-\frac{5}{2}\ m \cdot s^{-1/2})t^{-3/2}$$

$$a_x(t_1) = (-\frac{5}{2}\ m \cdot s^{-1/2})(5s)^{-3/2} = -\frac{1}{2\sqrt{5}}\ m \cdot s^{-2}$$

Acceleration

Example

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ii) Find the average acceleration between $t_1 = 5s$ and $t_2 = 15s$.

$$v_x(t) = \dot{x}(t) = \frac{(5\ m \cdot s^{-1/2})}{\sqrt{t}}\text{. So,}$$

$$v_x(t_1) = v_x(5s) = \sqrt{5}\ m/s\text{, and } v_x(t_2) = v_x(15s) = \sqrt{\frac{5}{3}}\ m/s\,.$$

$$a_{x,avg} = \frac{v_x(t_2) - v_x(t_1)}{t_2 - t_1} = \frac{(\sqrt{\frac{5}{3}} - \sqrt{5})\ m/s}{(15-5)s} = -\,0.0945\ m \cdot s^{-2}$$

Physical dimensions of velocity and acceleration

Both the velocity and the acceleration concepts are defined from the notions of distance and time interval: they are called ***derived quantities***. That is because they can be defined from the more primitive (undefinable) concepts that are space and time.

A natural consequence is that the dimensions (and units) of derived quantities can be expressed in terms of the dimensions of the primitive quantities they are defined from.

$$[v_x] = [\dot{x}] = L \cdot T^{-1}$$
$$[a_x] = [\dot{v}_x] = [\ddot{x}] = L \cdot T^{-2}$$

Reminder on calculus: primitive functions

Let us consider a continuous function $f(x)$ in $\mathbb{R}$. We call a ***primitive function*** F of f a function whose derivative is f

i.e. such that $$F'(x) = f(x)$$

- The primitive function of a function is not unique. If $F(x)$ is a primitive function of $f(x)$ then $F(x) + c$, for a constant number c, is a primitive function of $f(x)$ as well.

To express all the primitive functions of $f(x)$, we write:

$$\int f(x)dx = F(x) + c, \text{ for a constant } c$$

(indefinite integral)

Reminder on calculus: primitive functions

Functions	Primitives
Cx^{α}	$\frac{C}{\alpha+1}x^{\alpha+1}$
$Ce^{\alpha x}$	$\frac{C}{\alpha}e^{\alpha x}$
$\frac{C}{x}$	$C\ln x$
$\cos \alpha x$	$\frac{1}{\alpha}\sin \alpha x$
$\sin \alpha x$	$-\frac{1}{\alpha}\cos \alpha x$
$\cosh \alpha x$	$\frac{1}{\alpha}\sinh \alpha x$
$\sinh \alpha x$	$\frac{1}{\alpha}\cosh \alpha x$

Reminder on calculus: primitive functions

Example

Determine all the solutions of the equation $\dfrac{df(x)}{dx} = x^3 + 1.$

Reminder on calculus: primitive functions

Example

Determine all the solutions of the equation $\dfrac{df(x)}{dx} = x^3 + 1.$

$$f(x) = \int (x^3 + 1)dx = \frac{x^4}{4} + x + c,$$

where c is an arbitrary constant.

Particle under constant velocity

Constant velocity

The simplest case of all, is that of a particle moving with constant velocity, i.e.

$$v_x(t) = constant = v_x$$

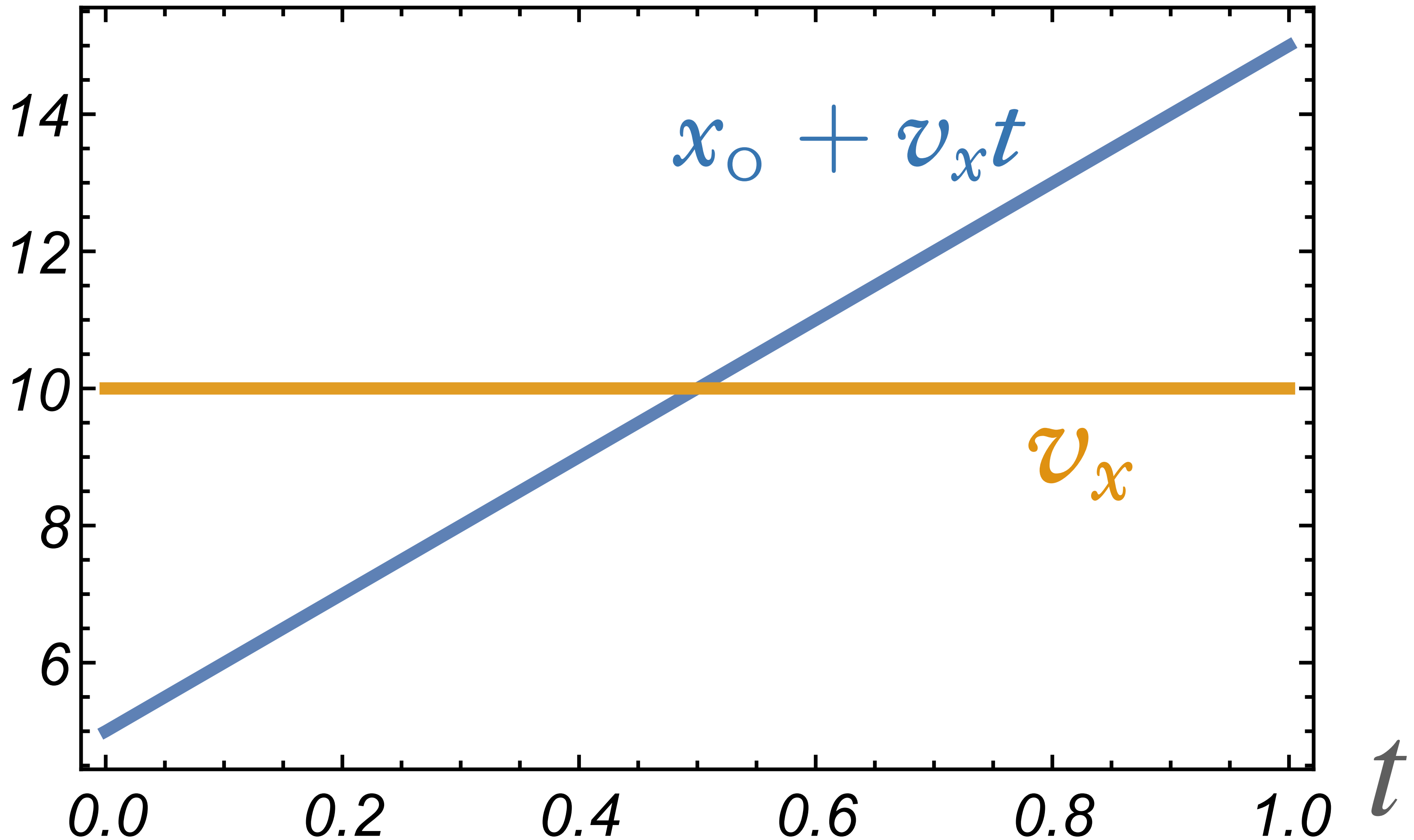
The position of the particle $x(t)$ satisfies the equation $\frac{dx(t)}{dt} = v_x$ which implies that $x(t) = v_x t + c$, for a constant $c \in \mathbb{R}$.

If the initial position at $t = 0$ is $x(0) = x_0$, we derive that $c = x_0$. Hence, the position of the particle is expressed as

$$x(t) = v_x t + x_0$$

Constant velocity

Graphical representation



Particle under constant acceleration

Constant acceleration

The second simplest case of kinematics is that of constant acceleration. It reads:

$$a_x(t) = constant = a_x$$

The velocity $v_x(t)$ satisfies the equation $\dfrac{dv_x(t)}{dt} = a_x$.

So, $v_x(t) = a_x t + c_1$, for a constant $c_1 \in \mathbb{R}$.

If the initial velocity at $t = 0$ is $v_x(0) = v_0$, we derive that $c_1 = v_0$.

Hence, the velocity of the particle is expressed as

$$v_x(t) = v_\circ + a_x t$$

Constant acceleration

We also know that $\dfrac{dx(t)}{dt} = v_x(t)$. So, for $v_x(t) = v_0 + a_x t$, we get

$\dfrac{dx(t)}{dt} = v_0 + a_x t$ and by integrating $x(t) = v_0 t + a_x \dfrac{t^2}{2} + c_2$,

for a constant $c_2 \in \mathbb{R}$.

If the initial position at $t = 0$ is $x(0) = x_0$, we derive that $c_2 = x_0$.

Hence, the position of the particle is expressed as

$$x(t) = x_0 + v_0 t + a_x \frac{t^2}{2}$$

Constant acceleration

Finally, the equations that describe the motion (*equations of motion*) of a particle moving with constant acceleration a_x are:

$$x(t) = x_0 + v_0 t + a_x \frac{t^2}{2}$$

$$v_x(t) = v_0 + a_x t$$

By combining these two equations we can write

$$x(t) = x_0 + \frac{1}{2}(v_0 + v_x(t))t$$

Constant acceleration

Example

A racing car goes from 0 to 60 mph in 3 seconds. Assuming it has constant acceleration, what is the distance it has traveled in that time in metres?

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For $t = 3s = \frac{3}{3600}h$, we have

$$v_x(t) = 60 \cdot 1609 \ m \cdot h^{-1} = 96540 \ m \cdot h^{-1} \quad \text{and} \quad v_0 = 0. \ \text{So,}$$

$$x(t) - x_0 = \frac{1}{2}(v_0 + v_x(t))t = \frac{1}{2}(96540m \cdot h^{-1})(\frac{3}{3600}h) = 40.225m$$