•• **[SSM]** (*a*) Estimate how close an approaching car at night on a flat, straight stretch of highway must be before its headlights can be distinguished from the single headlight of a motorcycle. (*b*) Estimate how far ahead of you a car is if its two red taillights merge to look as if they were one.

Picture the Problem Assume a separation of 1.5 m between typical automobile headlights and tail lights, a nighttime pupil diameter of 5.0 mm, 550 nm for the wavelength of the light (as an average) emitted by the headlights, 640 nm for red taillights, and apply the Rayleigh criterion.

(*a*) The Rayleigh criterion is given by Equation 33-25:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

where D is the diameter of the aperture of the eye.

The critical angular separation is also given by:

$$\alpha = \frac{d}{L}$$

where d is the separation of head lights (or tail lights) and L is the distance to approaching or receding automobile.

Equate these expressions for α_c to obtain:

$$\frac{d}{L} = 1.22\frac{\lambda}{D} \Rightarrow L = \frac{Dd}{1.22\lambda}$$

Substitute numerical values and evaluate L:

$$L = \frac{(5.0\text{ mm})(1.5\text{ m})}{1.22(550\text{ nm})} \approx \boxed{11\text{ km}}$$

(*b*) For red light:

$$L = \frac{(5.0\text{ mm})(1.5\text{ m})}{1.22(640\text{ nm})} \approx \boxed{9.6\text{ km}}$$

Explain why the ability to distinguish the two headlights of an oncoming car, at a given distance, is easier for the human eye at night than during daylight hours. Assume the headlights of the oncoming car are on during both daytime and nighttime hours.

Determine the Concept The condition for the resolution of the two sources is given by Rayleigh's criterion: $\alpha_c = 1.22\lambda/D$ (Equation 33-25), where α_c is the critical angular separation, D is the diameter of the aperture, and λ is the wavelength of the light coming from the objects, in this case headlights, to be resolved. Because the diameter of the pupils of your eyes are larger at night, the critical angle is smaller at night, which means that at night you can resolve the light as coming from two distinct sources when they are at a greater distance.

It is claimed that the Great Wall of China is the only man made object that can be seen from space with the naked eye. Check to see if this claim is true, based on the resolving power of the human eye. Assume the observers are in low-Earth orbit that has an altitude of about 250 km.

Picture the Problem We'll assume that the diameter of the pupil of the eye is 5.0 mm and use the best-case scenario (the minimum resolvable width varies directly with the wavelength of the light reflecting from the object) that the wavelength of light is 400 nm (the lower limit for the human eye). Then we can use the expression for the minimum angular separation of two objects than can be resolved by the eye and the relationship between this angle and the width of an object and the distance from which it is viewed to support the claim.

Relate the width w of an object that can be seen at an altitude h to the critical angular separation α_c:

$$\tan\alpha_c = \frac{w}{h} \Rightarrow w = h\tan\alpha_c$$

The minimum angular separation α_c of two point objects that can just be resolved by an eye depends on the diameter D of the eye and the wavelength λ of light:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

Substitute for α_c in the expression for w to obtain:

$$w = h\tan\left(1.22\frac{\lambda}{D}\right)$$

Substitute numerical values and evaluate w_{min} for an altitude of 250 km:

$$w_{min} = (250\,\text{km})\tan\left(1.22\left(\frac{400\,\text{nm}}{5.0\,\text{mm}}\right)\right) \approx 24\,\text{m}$$

This claim is probably false. Because the minimum width that is resolvable from low-Earth orbit (250 km) is 24 m and the size of the Great Wall is 5 to 8 m high and 5 m wide, this claim is likely false. However, it is easily seen using binoculars, and pictures can be taken of it using a camera. This is because both binoculars and cameras have apertures that are larger than the pupil of the human eye. (The Chinese astronaut Yang Liwei reported that he was not able to see the wall with the naked eye during the first Chinese manned space flight in 2003.)

•• [SSM] Estimate the maximum distance at which a binary star system could be resolvable by the human eye. Assume the two stars are about fifty times farther apart than Earth and Sun are. Neglect atmospheric effects. (A test similar to this "eye test" was used in ancient Rome to test for eyesight acuity before entering the army. A person who had normal eyesight could just barely resolve two well-known stars that appear close in the sky. Anyone who could not tell there were two stars failed the test.)

Picture the Problem Assume that the diameter of a pupil at night is 5.0 mm and that the wavelength of light is in the middle of the visible spectrum at about 550 nm. We can use the Rayleigh criterion for the separation of two sources and the geometry of the Earth-binary star system to derive an expression for the distance to the binary stars.

If the distance between the binary stars is represented by d and the Earth-star distance by L, then their angular separation is given by:

$$\alpha = \frac{d}{L}$$

The critical angular separation of the two sources is given by the Rayleigh criterion:

$$\alpha_c = 1.22\frac{\lambda}{D}$$

For $\alpha = \alpha_c$:

$$\frac{d}{L} = 1.22\frac{\lambda}{D} \Rightarrow L = \frac{Dd}{1.22\lambda}$$

Substitute numerical values and evaluate L:

$$L = \frac{(5.0\text{ mm})(50)(1.5\times10^{11}\text{ m})}{1.22(550\text{ nm})}$$

$$\approx 5.59\times10^{13}\text{ km}\times\frac{1\,c\cdot\text{y}}{9.461\times10^{15}\text{ m}}$$

$$\boxed{5.9\,c\cdot\text{y}}$$

• Light of wavelength 500 nm is incident normally on a film of water 1.00 μm thick. (*a*) What is the wavelength of the light in the water? (*b*) How many wavelengths are contained in the distance $2t$, where t is the thickness of the film? (*c*) What is the phase difference between the wave reflected from the top of the air–water interface and the wave reflected from the bottom of the water–air interface in the region where the two reflected waves superpose?

Picture the Problem The wavelength of light in a medium whose index of refraction is n is the wavelength of the light in air divided by n. The number of wavelengths of light contained in a given distance is the ratio of the distance to the wavelength of light in the given medium. The difference in phase between the two waves is the sum of a π phase shift in the reflected wave and a phase shift due to the additional distance traveled by the wave reflected from the bottom of the water–air interface.

(*a*) Express the wavelength of light in water in terms of the wavelength of light in air:

$$\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n_{\text{water}}} = \frac{500\,\text{nm}}{1.33} = \boxed{376\,\text{nm}}$$

(*b*) Relate the number of wavelengths N to the thickness t of the film and the wavelength of light in water:

$$N = \frac{2t}{\lambda_{\text{water}}} = \frac{2(1.00\,\mu\text{m})}{376\,\text{nm}} = \boxed{5.32}$$

(*c*) Express the phase difference as the sum of the phase shift due to reflection and the phase shift due to the additional distance traveled by the wave reflected from the bottom of the water–air interface:

$$\delta = \delta_{\text{reflection}} + \delta_{\text{additional distance traveled}}$$
$$= \pi + \frac{2t}{\lambda_{\text{water}}} 2\pi = \pi + 2\pi N$$

Substitute for N and evaluate δ:

$$\delta = \pi\,\text{rad} + 2\pi(5.32\,\text{rad}) = 11.64\pi\,\text{rad}$$
$$= \boxed{11.6\pi\,\text{rad}}$$

or, subtracting 11.64π rad from 12π rad,

$$\delta = 0.4\pi\,\text{rad} = \boxed{1.1\,\text{rad}}$$