Algebra – Tutorial week 6

T6.1. Use the Euclidean algorithm to find the greatest common divisor of the polynomials $2x^3 - 3x^2 - x - 2$ and $x^3 - x^2 - x - 2$.

Then use the greatest common divisor to partially factorise each polynomial.

T6.2. Factorise the following polynomials as far as you can using only real numbers:

(a) $2x^6 - 128$;
(b) $x^4 - 9$;
(c) $x^{16} - 1$.

T6.3*. Consider the polynomials $f_t(x) = x^3 + tx^2 - x + (t+1)$ and $g_t(x) = x^2 - tx + 1$.

In principle they could be viewed as polynomials in two indeterminates x and t, but here we want to view them as polynomials in just x, with coefficients depending on a *parameter* t (so the coefficients are functions of t). To make that clear we have used the notation $f_t(x)$ rather than $f(x,t)$.

(a) Divide $f_t(x)$ by $g_t(x)$ with remainder (by the usual long division).
(b) Find any values of the parameter t for which the remainder is a constant (so it does not involve x).
(c) Find any values of the parameter t for which $g_t(x)$ divides $f_t(x)$.